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Solution of One Class of Systems of Stochastic Differential Equations

O. V. Zakharova[1*]

[1]*Ufa State Aviation Technical University, ul. K. Marksa 12, Ufa, 450000 Russia*

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Abstract—In this paper we prove that instead of solving a class of systems of stochastic differential equations with a multidimensional Wiener process one can solve a system of total differential equations. The latter system admits an application of classical methods. This fact enables one to solve the initial system explicitly.

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INTRODUCTION

Stochastic differential equations include equations in the form

$$\eta(t) - \eta(0) = \int_0^t a(s,\eta(s)) * dw(s) + \int_0^t b(s,\eta(s))ds,$$

where $\int\limits_0^t a(s,\eta(s)) * dw(s)$ is the stochastic Stratonovich integral with respect to the Wiener process $w(s)$. We understand explicit formulas for solutions to stochastic differential equations (SDE in what follows) as an ordinary differential equation or a chain of such equations that allow one to find a solution to the initial SDE. Prior to paper [1] explicit formulas for solutions were known only for a rather narrow class of equations [2, 3]. In paper [1] explicit formulas were constructed for SDE and for systems of such equations with one-dimensional Wiener process. In this paper we continue the studies described in papers [1], [4], and [5]. The approach used in the mentioned papers has a specific feature, namely, the application of symmetric integrals introduced in [4]. In the latter paper one constructs symmetric integrals for an arbitrary continuous function, in particular, for trajectories of the Wiener process. The goal of this paper is to obtain (with the help of symmetric integrals) explicit formulas for a solution to one class of systems of SDE with a multidimensional Wiener process.

Definition. Consider partitions $T_n,\ n \in N,$ of the segment $[0,t] : T_n = \{t_k^{(n)}\},\ 0 = t_0^{(n)} \le t_1^{(n)} \le \dots \le t_k^{(n)} \le \dots \le t_{m_n}^{(n)} = t,\ n \in N,$ such that $T_n \subset T_{n+1},\ n \in N,$ and $\lambda_n = \max\limits_k |t_k^{(n)} - t_{k-1}^{(n)}| \to 0$ as $n \to \infty$. Denote by $X^{(n)}(s)$, $s \in [0,t]$, a polygonal line constructed by a continuous function $X(s)$ that corresponds to the partition T_n, $\Delta t_k^{(n)} = t_k^{(n)} - t_{k-1}^{(n)}$, $\Delta X_k^{(n)} = X(t_k^{(n)}) - X(t_{k-1}^{(n)})$. We understand a *symmetric integral* as

$$\int_0^t f(s,X(s)) * dX(s) = \lim_{n\to\infty} \sum_k \frac{1}{\Delta t_k^{(n)}} \int_{[t_{k-1}^{(n)},\ t_k^{(n)}]} f(s,X^{(n)}(s))ds\Delta X_k^{(n)},$$

provided that the limit in the right-hand side of the equality exists and is independent of the choice of the sequence of partitions T_n, $n \in N$. The symmetric integral is a determinate analog of the stochastic

*E-mail: dear_hunny@mail.ru.