Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 43, No. 1, pp. 65–71, 1999 *UDC 519.63*

INVESTIGATION OF CONVERGENCE OF DIFFERENCE SCHEME FOR 3D EQUATIONS OF THE VISCOUS LIQUID DYNAMICS

Ye.M. Fedotov

Construction and investigation of difference schemes for nonstationary equations of the Mathematical Physics represent a significant interest from both practical and theoretical points of view. One of most interesting and important aspects of the theory of difference schemes is the investigation of their correctness. The general theory of correctness of operator-difference schemes (ODS) constructed in [1]–[4] allows to reduce the investigation of concrete difference schemes in the linear case to verification of the properties of the participating grid operators. The theory of difference schemes for nonlinear problems is less developed. We can note here the papers [5]–[12], where sufficiently general conditions on the difference operators are formulated, ensuring both resolvability and stability of some classes of difference equations. Let us also note the papers [13]–[17], where estimates of the accuracy of some types of difference schemes were obtained for the problems of one and two dimensional gas dynamics. In the present article, using results on the correctness of a system of two-layer operator-difference schemes, which were obtained in [12], we investigate the convergence of a grid scheme for 3D Lagrange system of equations of the dynamics of a viscous compressible fluid in the variables specific volume–velocity–entropy under general assumptions on the state equations.

Consider the problem on the flow of a viscous liquid which occupies the initial volume $\overline{\Omega} = \{0 < \overline{x}_i < 1,\ i = \overline{1,m}\}$, $m = 2,3$, with the boundary Γ. The flow of the fluid is described by the system of equations

$$\rho_0 \frac{d\eta}{dt} - |J| \nabla \cdot \vec{v} = 0, \tag{1}$$

$$\rho_0 \frac{d\vec{v}}{dt} + |J| (\nabla p - (\lambda + \mu) \nabla \operatorname{div} \vec{v} - \mu \Delta \vec{v}) = \rho_0 \vec{f}, \tag{2}$$

$$\rho_0 T \frac{ds}{dt} = |J| ((\lambda + \mu)(\operatorname{div} \vec{v})^2 + \mu \nabla v_i \cdot \nabla v_i), \tag{3}$$

$$\frac{d\vec{x}}{dt} = \vec{v}, \quad t > 0, \tag{4}$$

where η, $\vec{v}$, and s are the specific volume, velocity, and entropy, respectively, λ and μ are the Lame coefficients, $|J| = \det(J)$, $J \equiv J(\vec{x}) = (\partial x_i / \partial \overline{x}_j)_{i,j=1}^m$ is the Jacobi matrix of the transition from reference Cartesian (Lagrange) system of coordinates $\vec{\overline{x}}$ to the Eulerian one, $p = -\partial \varepsilon / \partial \eta$ is the pressure, $T = \partial \varepsilon / \partial s$ is the temperature, and $\varepsilon = \varepsilon(\eta, s)$ is the thermodynamic potential.

We write out both the boundary and initial conditions for equations (1)–(4) in the form

$$\begin{gathered} \vec{v} = 0, \quad \overline{x} \in \Gamma, \\ (\eta, \vec{v}, s) = (\eta_0, \vec{v}_0, s_0), \quad \vec{x} = \vec{x}_0 \equiv \vec{\overline{x}}, \quad t = 0, \quad \overline{x} \in \Omega. \end{gathered} \tag{5}$$

Supported by Russian Foundation for Basic Research (project 95-01-00400).