

MATRIX NONLINEAR SCHRÖDINGER EQUATION AND THE CORRESPONDING EQUATION HIERARCHY

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In the present article we consider a family of equations

$$\frac{\partial G}{\partial t} - \frac{\partial F_N}{\partial x} + [G, F_N] = 0; \quad N = 1, 2, 3, \dots, \quad (0.1)$$

where $[\cdot, \cdot]$ is commutant symbol, i. e., $[G, F_N] = GF_N - F_N G$. The matrices $G(x, t, z)$ and $F_N(x, t, z)$ have the order $2m \times 2m$ and are the polynomials in z of the respective degrees 1 and N . For $N = 2$ and a certain reduction, equation (0.1) coincides with well-known matrix nonlinear Schrödinger equation, while for $N = 3$ — with the matrix modified Korteweg–de Vries equation. The family of equations (0.1) forms so-called hierarchy. In the present article we prove that equations (0.1) are differential equations; we derive recurrent formulas for construction of $F_N(x, t, z)$. On this path we find an infinite set of polynomial conservation laws for equations (0.1). For the scalar case ($m = 1$), close results are known (see [1], [2]). Further we use results obtained for construction of asymptotics of the Weil–Titchmarsh matrix function $v_N(t, z)$ as $z \rightarrow \infty$ of canonical system of differential equations of the form (see [3], [4])

$$\frac{\partial W}{\partial x} = G(x, t, z)W. \quad (0.2)$$

This result is also new in the scalar case. In the article we introduce the classes of regularity \mathcal{P}_T and derive the theorem on existence and uniqueness of the solution of equation (0.1) in the class \mathcal{P}_T in the domain $0 \leq x < \infty$, $0 \leq t \leq T$. In addition, we suppose that only initial data is known.

It is of interest to note that for the stationary linear problem

$$\frac{dW}{dx} = G(x, 0, z)W \quad (0.3)$$

the asymptotic of the corresponding Weil–Titchmarsh function $v(z)$ can also be found. In doing so, we use step-by-step nonstationary nonlinear equations of the hierarchy.

Thus, here we reveal the interconnections between hierarchy equations, conservation laws, and asymptotic coefficients of the Weil–Titchmarsh function.

1. Construction of equation hierarchy

1. Let the matrix functions $G(x, t, z)$ and $F_N(x, t, z)$ have the form

$$G(x, t, z) = g_1(x, t) + zg_0(x, t), \quad (1.1)$$

$$F_N(x, t, z) = f_N(x, t) + zf_{N-1}(x, t) + \dots + z^N f_0(x, t). \quad (1.2)$$

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