Russian Mathematics (Iz. VUZ)
Vol. 45, No. 7, pp.55–61, 2001

Izvestiya VUZ. Matematika
UDC 517.988.67

PARAMETERIZATION OF SIMPLE BRANCHING SOLUTIONS OF COMPLETE RANK AND ITERATIONS IN THE NONLINEAR ANALYSIS

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Dedicated to 70-th anniversary of Professor V.A. Trenogin's birth

Introduction

The role of the geometry of planes, diagrams, and Newton's polyhedrons is well-known (see [1], [2]) in the asymptotical analysis of the theory of branching (see [1]–[5] and other papers). In the construction of branching solutions by iteration methods, both explicit and implicit parameterizations were used (see [6]–[12]), and in complex cases (see [10], [13]–[15]) — the geometrical constructions indicated above. In [6], [7], and [9]–[12], the rigid constraints ensuring access to the principal part of the equation of branching (e. b.) already at the first iteration were of a decisive importance for the convergence of methods. In particular cases a method for construction of the initial approximation and the choice of parameterization of the sought-for branch were indicated. In [6], [7], [9], [11], and [12] the uniformization parameter was chosen directly from the form of the Newton diagram of the coefficients of the projection QF. Therefore on each iteration it was necessary to solve one linear equation. Iteration methods for more complex situations in [6]–[12] were not developed, because the ways for the parameter of uniformization of branches to be chosen and iteration process to be organized in general case were not clear.

In [13], [14] it was shown that both the essential expansion of the class of equations and creation of more flexible algorithms are possible by means of application of N-step methods, in which on each iteration N linear equations are solved and different parameters of uniformization are applied in accordance with [2]. In [13] an N-step iteration method was suggested which converges in a neighborhood of the branching point under constraints which are weaker than those of other methods. The method possesses the advantages of both explicit and implicit parameterization (see [12]), since it gives us a certain freedom in the choice of the uniformization parameter. However, the iteration scheme suggested on the basis of this method in [13] (see § 2 there), requires, in the general case, a preliminary construction of an element $\widehat{x}^0$ from the subspace $E_1^{\infty-n}$, which is used at the initial approximation. A method for construction of the element $\widehat{x}^0$ was given in [13] (see lemma 2 there). Nevertheless, the preliminary computation of $\widehat{x}^0$ makes both the computations and the proper iteration scheme more complicated.

The objective of this article is not a simple modification and expansion of the possibilities of iteration methods in [11], [12], [16], but also deduction of iteration formulas convenient for the theory of approximate methods and its applications. A more convenient iteration scheme with simplified choice of the initial approximation is suggested.

Supported by INTAS, grant 2000-15.