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APPROXIMATION OF BOUNDARY VALUE PROBLEMS OF ELLIPTIC TYPE WITH A SPECTRAL PARAMETER AND DISCONTINUOUS NONLINEARITY

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Introduction. Discontinuous nonlinearities in integral and differential equations often emerge as idealizations of continuous nonlinearities which have sections of the fast growth with respect to a phase variable. It is convenient to assume that continuous nonlinearities depend on a small parameter ε and at the limit for $\varepsilon \to 0$ one obtains a discontinuous nonlinearity. It emerges the natural question about the proximity of solutions of the approximating equation and those of the limit problem. The necessity to investigate this question is mentioned in [1]. For elliptic boundary value problems with discontinuous nonlinearities this problem is studied in [2] under the assumption that the nonlinearity of the equation is bounded and the differential part, together with the boundary condition, generates a coercitive operator.

In this paper, we consider resonance elliptic boundary value problems with a spectral parameter and bounded discontinuous nonlinearity. The resonance property of the boundary value problem means that the kernel of the differential operator with the corresponding boundary value condition is nonzero. In [3], by the variational method the existence of a ray of positive proper values for such problems is established. The approximating problem is derived from the initial one by small perturbations of the spectral parameter and continuous with respect to the phase variable approximations of the discontinuous nonlinearity. The aim of this paper is to establish under certain conditions the convergence of solutions of approximating problems to those of the initial one.

1. Problem definition. Let Ω be a bounded domain in $\mathbf{R}^n$ with the boundary Γ of the class $\mathbf{C}_{2,\alpha}$, $0 < \alpha \le 1$ ([4], p. 23); let the hypersurfaces

$$S_i = \{(x,u) \in \mathbf{R}^{n+1} : u = \varphi_i(x),\ x \in \overline{\Omega}\},$$

$\varphi_i \in \mathbf{W}_q^2(\Omega)$, $q > n$, $i = \overline{1,m}$, be pairwise disjoint. For definiteness, we assume that $\varphi_i(x) < \varphi_{i+1}(x)$ for any $x \in \overline{\Omega}$ and $i = \overline{1,m-1}$. Since $q > n$, we have $\varphi_i(x) \in \mathbf{C}_1(\overline{\Omega})$ ([5], p. 74). Hence and from the latter inequality we obtain the existence of $d > 0$ such that for any $x \in \overline{\Omega}$ segments $[\varphi_i(x) - d,\ \varphi_i(x) + d]$, $i = \overline{1,m}$, are pairwise disjoint.

Surfaces S_i, $i = \overline{1,m}$, break the domain $D = \Omega \times \mathbf{R}$ into disjoint subdomains

$$\begin{aligned} D_0 &= \{(x,u) \in D : u < \varphi_1(x)\}, \\ D_i &= \{(x,u) \in D : \varphi_i(x) < u < \varphi_{i+1}(x)\},\ i = \overline{1,m-1}, \\ D_m &= \{(x,u) \in D : u > \varphi_m(x)\}. \end{aligned}$$

On $\overline{D_i}$ the Caratheodory functions ([6], p. 148) $g_i(x,u)$ are defined such that for almost all $x \in \Omega$,

$$|g_i(x,u)| \le a(x)\ \ \forall u,\ \ (x,u) \in \overline{D_i}, \tag{1}$$