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ON CURVATURE OF CR-SUBMANIFOLDS OF MAXIMAL CR-DIMENSION IN COMPLEX PROJECTIVE SPACE

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Let M be an n-dimensional CR-submanifold of CR-dimension $\frac{n-1}{2}$ in complex projective space, i. e., let M be a real submanifold of complex projective space such that the maximal holomorphic subspace of tangent space of M at x is $(n-1)$-dimensional for any $x \in M$. Then M is necessarily odd-dimensional, and there exists a unit vector field ξ normal to M such that $JT(M) \subset T(M) \oplus \operatorname{span}\{\xi\}$. We study such submanifolds under the assumption that the distinguished vector field ξ is parallel with respect to the normal connection, and we determine a sufficient condition for such a submanifold to be an open subset of a geodesic sphere.

0. Introduction

The study of real hypersurfaces in complex projective space has been an active field over many years. Although the complex projective space might be regarded as the simplest one after the spaces of constant curvature, it imposes significant restrictions on the geometry of its hypersurfaces. For instance, there exist neither umbilic nor Einstein real hypersurfaces in complex projective space [1]. R. Takagi's classification [2] of homogeneous real hypersurfaces of complex projective space was important in its own right, but it also identified a whole list of hypersurfaces, gave them names and brought them into focus. Many geometricians began to study them deriving new characterizations of various subclasses of R. Takagi's classification list (see [1], [3], [4], [5]). Namely, characterizations of certain subclasses have been implemented according to properties of the shape operator, Ricci tensor, and other geometric objects. More details and comprehensive bibliography can be found in [6].

In the present paper we continue the above-mentioned study and find another sufficient condition for an n-dimensional real submanifold of codimension p with maximal holomorphic tangent subspace in complex projective space to be an open subset of a geodesic sphere. It is well-known that a real hypersurface of a complex projective space has an almost contact metric structure induced from the complex structure. Moreover, in Section 1, we prove that a geodesic hypersphere in a complex projective space is a Sasakian manifold, we compute its curvature tensor and remark that it has the form of the curvature tensor of a Sasakian space form [7].

Therefore, if the curvature tensor R of a submanifold M is given by

$$\begin{aligned}R(X,Y)Z = {}&\alpha\{g(Y,Z)X - g(X,Z)Y\} + \\ &+ \beta\{g(FY,Z)FX - g(FX,Z)FY - 2g(FX,Y)FZ\} + \\ &+ \gamma\{u(Y)u(Z)X - u(X)u(Z)Y + u(X)g(Y,Z)U - u(Y)g(X,Z)U\},\end{aligned}$$

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