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The Inverse Problem for a Mixed-Type Parabolic-Hyperbolic Equation in a Rectangular Domain

K. B. Sabitov[1]* and E. M. Safin[2]**

[1]*Academy of Sciences of the Republic of Bashkortostan, Sterlitamak Branch, ul. Odesskaya 68, Sterlitamak, 453103 Russia*

[2]*Institute of Mathematics with Computing Center, Ufa Scientific Center, Russian Academy of Sciences ul. Chernyshevskoko 112, Ufa, 450077 Russia*

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Abstract—In this paper with the help of the spectral method we obtain a criterion for the unique solvability of the inverse problem for a mixed-type parabolic-hyperbolic equation in a rectangular domain. This problem implies the search of the unknown right-hand side.

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1. Problem definition. Let us consider a mixed-type parabolic-hyperbolic equation in the form

$$Lu = \begin{cases} u_t - u_{xx} = f(x), & t > 0; \\ u_{tt} - u_{xx} = f(x), & t < 0, \end{cases} \tag{1}$$

in a rectangular domain $D = \{(x,t) \mid 0 < x < 1,\ -\alpha < t < \beta\}$; here α and β are given positive real numbers.

Problem. *In the domain D find functions $u(x,t)$ and $f(x)$ that satisfy the conditions*

$$u(x,t) \in C(\overline{D}) \cap C^1(D) \cap C^{2,1}_{x,t}(D_+ \cup \{t=\beta\}) \cap C^2(D_- \cup \{t=-\alpha\}), \tag{2}$$

$$f(x) \in C(0,1), \tag{3}$$

$$Lu(x,t) \equiv f(x), \quad (x,t) \in D_+ \cup D_-, \tag{4}$$

$$u(0,t) = u(1,t) = 0, \quad -\alpha \le t \le \beta, \tag{5}$$

$$u(x,-\alpha) = \psi(x), \quad 0 \le x \le 1, \tag{6}$$

$$u(x,\beta) = \varphi(x), \quad 0 \le x \le 1, \tag{7}$$

where $\psi(x)$ and $\varphi(x)$ are given sufficiently smooth functions, $\varphi(0) = \varphi(1) = 0$, $\psi(0) = \psi(1) = 0$, $D_+ = D \cap \{t > 0\}$, $D_- = D \cap \{t < 0\}$.

By now the most complete results have been obtained in studying direct problems for mixed-type equations. For example, papers [1–7] are dedicated to boundary-value problems for equations of the mixed parabolic-hyperbolic type. At the same time, there are almost no papers connected with solving inverse problems for mixed-type equations. Inverse problems for certain types of partial differential equations were studied by many authors [8–12].

In this paper we establish a criterion for the unique existence of a solution to the inverse problem (1)–(7) by the method of spectral analysis [13].

*E-mail: Sabitov_fmf@mail.ru.
**E-mail: Eldar4u@rambler.ru.