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INVESTIGATION OF POSITIVE BY CURVATURE SOLUTIONS OF SYSTEM OF EQUILIBRIUM EQUATIONS OF A CLOSED CYLINDRICAL SHELL

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1. Introduction

We investigate convex forms of equilibrium of a closed infinitely long elastic cylindrical shell when the bending stiffness tends to zero. The following assumptions simplify the investigation: external loads are tangent, outer moments are absent, the median surface is non-stretched, the initial form of the shell is circular.

Equations of static equilibrium of the cylindrical shell in these assumptions are as follows (see [1]):

$$\frac{dT}{ds} + k(s)N(s) = 0, \quad \frac{dN}{ds} - k(s)T(s) - f(s) = 0, \quad \frac{dM}{ds} - N(s) = 0. \tag{1}$$

Here $T(s)$, $N(s)$ are the tangent and cutting forces, respectively, at the point of the contour of deformed cylinder with the arc coordinate s; $f(s)$ the linear density of normally acting load; in the motion along the increase of s the external normal is directed to the right; $M(s)$ is the bending moment connected with the curvature of the directrix $k(s)$ via the Loeve formula,

$$M(s) = D(k(s) - k_u(s)), \tag{2}$$

D is the bending stiffness, $k_u(s)$ the curvature of the contour of the cylinder in the non-deformed state. Let s vary within the limits of the interval $[0,2]$, then $k_u(s) = \pi$. In addition to (1) and (2) we impose the conditions of closedness

$$\int_0^2 \cos\Big(\int_0^s k(\xi)d\xi\Big)ds = \int_0^2 \sin\Big(\int_0^s k(\xi)d\xi\Big)ds = 0 \tag{3}$$

and the condition upon the rotation angle of the tangent within the complete path-tracing

$$\int_0^2 k(\xi)d\xi = 2\pi. \tag{4}$$

We consider only classical solutions of problem (1), (2); we assume that $k(s) \in C^2[0,2]$. From the last equation in (1), by excluding M by means of (2), we get $N(s) = Dk'(s)$, and then from the first equation in (1) it follows that the value

$$\omega = \frac{T(s)}{D} + \frac{1}{2}k^2(s) \tag{5}$$

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