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ECONOMIC ITERATION SCHEMES OF REALIZATION OF FINITE ELEMENT METHOD FOR STATIONARY BOUNDARY VALUE PROBLEMS OF THE MATHEMATICAL PHYSICS

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In the article we suggest and investigate economic iteration methods for solving stationary problems of the mathematical physics with the use of the Galyorkin method (see [1], [2]). In [2] for the first time the splitting-up method in combination with the finite element method was suggested for two-dimensional parabolic convection–diffusion equations. This idea was further developed in creation of methods for solving a wider class of nonstationary problems (see [3]–[5]). However, we should note that the construction of economic difference schemes on the basis of the summary approximation method (component-wise splitting) (see [2], [3]), when the initial operator is decomposed into four one-dimensional nonnegative operators, leads us to constraints on coefficients of the initial equation, which are more rigid than the ellipticity condition. In addition, in our opinion, in using this method essential difficulties arise related to, first, constraints on the choice of steps of grid for ensuring the stability of the algorithm, and, second, the presence of additional requirements on the configuration of the finite element.

Within the development of works [6], [7] in [8]–[10] a multicomponent version of the alternating direction method was suggested, which possesses absolute stability in decomposing the operator into an arbitrary number of, generally speaking, non-commuting operators and which is related to the complete approximation spitting methods. In [11], [12], on the basis of this method, economic algorithms for solving finite element difference schemes (the quantity of decomposition is four or six) were constructed; in addition, in these works the non-intrinsic constraints arising in using the splitting method were removed.

Let us note that, in solving stationary problems of the mathematical physics, the difficulties arising in development of economic iteration methods grow significantly. In particular, the mentioned summary approximation method is practically inapplicable as an iteration establishing method, because it requires an essential decrease of iteration parameter for attainment of the desired accuracy. The presence of mixed derivatives in the initial differential operator also does not allow to use effectively the classical alternating direction method which requires the commutativity of decomposition operators. In this article we construct and study iteration schemes of multicomponent alternating direction method in combination with the finite element method for solving stationary equations of the convection–diffusion type. The results obtained in this article allow us to expand essentially the domain of applicability of the mentioned class of methods, which earlier were used only in the capacity of the finite-difference iteration schemes for elliptic boundary value problems (see [13]–[15]).

In a domain G with the boundary Γ, $G \subset \mathbf{R}^2$ is a bounded simply connected domain, $\Gamma \in \mathbf{C}^2$,