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A Certain Fredholm Partial Integro-Differential Equation of the Third Order

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Abstract—We study the unique solvability of the initial value problem for a nonlinear partial Fredholm integro-differential equation of the third order with a degenerate kernel. First, we adapt the degenerate kernel method developed for a partial Fredholm integro-differential equation of the second kind for solving Fredholm integro-differential equations of the third order. After solving the corresponding system of algebraic equations, we obtain a Volterra integral equation of the second kind. Then we apply the method of successive approximations combined with the method of contractive maps.

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1. THE PROBLEM

The mathematical modeling of many real-life processes often leads to initial and boundary-value problems for equations that have no analogs in classical mathematical physics. The theory of initial and boundary-value problems for partial differential equations due to its practical value is now one of the most important branches of the theory of differential equations. Problems of the gas dynamics, the elasticity theory, and the theory of plates and hulls lead to partial differential equations of higher orders [1].

One studies partial differential equations of the third order when solving problems of the theory of nonlinear acoustics and the hydrodynamic theory of cosmic plasma. Models of fluid filtration in porous media can often be reduced to differential equations of the third order [2]. Moreover, problems of wave propagation in weakly dispersive media and in cold plasma, as well as magnetohydrodynamics problems and some other ones can also be reduced to partial differential equations of the third order. The latter are studied in many papers (e.g., [3–10]).

In this paper, we propose a technique for studying the initial value problem for a nonlinear Fredholm partial integro-differential equation of the third order with a degenerate kernel. In a domain $\Omega \equiv \Omega_T \times R^+$ we consider the following Fredholm integro-differential equation:

$$\frac{\partial}{\partial x}\left(\frac{\partial^2 u(t,x)}{\partial t^2} - \lambda \int_0^T K(t,s)\frac{\partial u(s,x)}{\partial s}ds\right) = f\left(x, \int_0^T \int_0^\infty H(s,y)u(s,y)dy\,ds\right) \tag{1}$$

subject to initial conditions

$$u_x(0,x) = \varphi_1(x),\ \ u_{tx}(0,x) = \varphi_2(x),\ \ x \in R^+, \tag{2}$$

$$u(t,0) = \psi(t),\ \ t \in \Omega_T, \tag{3}$$

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