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A Modified Inverse Boundary-Value Problem for an Airfoil Located Close to a Rectilinear Barrier

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In this paper we consider the inverse boundary-value problem for an airfoil located close to a rigid rectilinear boundary. We obtain formulas for a solution of the stated problem and propose a simple technique which allows one to ensure the closedness of the desired airfoil. We consider the definition of parameters of the function which conformly maps the image of the flow domain in the plane of the complex potential into the canonical one.

1. Let an airfoil L_z situated in the plane $z = x + iy$ be flowed around by a nonviscous incompressible fluid. Assume that the stream is stable, confined by the straight line $y = -p$, $p = \text{const} > 0$, and has the rate $v_\infty e^{i\pi}$, $v_\infty > 0$ at infinity.

Let s stand for the arc abscissa of a point of L_z calculated from the stream bifurcation point A at the airfoil L_z in such a direction that the flow domain stays at the right. Let l be the perimeter of the airfoil L_z; let $s = s_B$ be the arc abscissa of the vanishing point B of the stream at the airfoil L_z, $0 < s_B < l$; let M and N be points of the airfoil L_z which correspond to certain values $s = s_M$ and $s = s_N$, respectively; $0 < s_M < s_B$, $s_B < s_N < l$ (M is a point of the upper surface of L_z); assume that the point A of the airfoil L_z coincides with the point $z = 0$.

Let $w(z) = \varphi + i\psi$ be the complex potential of the flow around L_z. We denote by v the module of the rate at the point z and by η the angle of the inclination of this rate to the real axis. We have $w'(z) = ve^{-i\eta}$. Let D_z stand for the flow domain. Put $\psi = 0$ on the line $y = -p$, $\psi = -Q = \text{const} < 0$ on the airfoil L_z.

On the arc AMB of the airfoil L_z we have $\varphi'_s = v$, on the rest part of the airfoil we have $\varphi'_s = -v$. Therefore for points of the arc AMB of the airfoil L_z we have $\varphi = \varphi(s) = \int\limits_0^s v\,ds$, $0 \le s \le s_B$. Let $\varphi_B = \varphi(s_B) = \int\limits_0^{s_B} v\,ds$. For values of the module of the rate v on the arc BNA we denote $\varphi_H = \int\limits_{s_B}^{l} v\,ds$. Let points M and N of the airfoil L_z be such that $\int\limits_0^{s_M} v\,ds = \frac{\varphi_B}{2}$, $\int\limits_{s_B}^{s_N} v\,ds = \frac{\varphi_H}{2}$. Due to the latter fact we have $\int\limits_{s_N}^{l} v\,ds = \frac{\varphi_H}{2}$.

Define the function $\varphi = \varphi(s)$ on the part BNA of the airfoil L_z as follows: $\varphi = \varphi(s) = \varphi_B - \int\limits_{s_B}^{s} v\,ds$, $s_B \le s < s_N$; $\varphi = \varphi(s) = \int\limits_s^l v\,ds$, $s_N \le s \le l$. Then the function $\varphi = \varphi(s)$ is continuous everywhere on L_z, except for the point N, in addition, $\varphi(s_N - 0) - \varphi(s_N + 0) = \varphi_B - \int\limits_{s_B}^{s_N} v\,ds - \int\limits_{s_N}^{l} v\,ds = \varphi_B - \varphi_H = \Gamma$ is the velocity circulation over the airfoil L_z.

Consider the solution of the following modified inverse boundary-value problem which is analogous to those studied in [1] (pp. 97–110, 292–299). The distribution of the value of the rate v along the airfoil L_z