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One Analytic Approach to the Solution of One-Dimensional Heat Conduction Problem with Free Boundaries

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In recent years the boundary-value heat conduction problems in domains with free boundaries become important both in theoretical and applied branches of physics and mathematics. Over the last years the number of papers dedicated to the solution of similar problems essentially increased [1–9]. The analytic approach to the solution of boundary-value heat transfer problems in systems with free boundaries is one of the most difficult problems in the modern mathematical physics. Since the position of the characteristic part of the domain depends on time, the classical methods for partial differential equations are inapplicable to this class of problems. The reason is that these methods do not allow one to correlate the solution of the heat conduction equation with the motion of the phase transfer boundary.

In this paper we propose to apply the method of degenerate hypergeometric transformations (DHGTs) [11, 12] to the solution of one nonstationary problem with phase transition for the heat transfer equation with a constant thermal conductivity coefficient. As an example we consider the freezing of a certain continuum. We obtain an analytic solution to the problem with special edge conditions. During the solution process we establish the law of motion of the interface of two phases. We consider the case, when the equation for the kernel of the integral transformation used in the solution of the problem is selfadjoint. This enables us to apply the method of expansion in characteristic functions. The definition of eigenvalues of the spectral problem is based on the known asymptotic expansion with the use of the technique presented in [10].

The essence of the method, solving the problem under consideration, consists in the following idea. Replacing a moving system of coordinates with a fixed one, one reduces the problem to the classical case with a fixed boundary, but variable coefficients occur. In the latter case one can apply the method of finite integral transformations, transferring the kernels of the latter to the space of images with the help of the developed algorithm [11].

Consider the application of the method of DHGTs [11, 12] to the heat transfer process with a phase transition during the freezing of some continuum under the influence of a flat source of cold. At the initial time moment the temperature of the medium is constant $t_0 > 0$ with $x \geq a\xi_0$. The temperature of the outer surface of the medium is $t_e < 0$. Frost zones ($k = 1$) and cold zones ($k = 2$) appear, and the phase transition boundary eventually moves into the interior of the medium. Let us state the mathematical model of this process for the one-dimensional scheme as follows:

$$\frac{\partial t_k(x,\tau)}{\partial \tau} = a_k \frac{\partial^2 t_k(x,\tau)}{\partial x^2}, \tag{1}$$

$$x \in D_1 = \{0 < x < \xi_1(\tau)\} \quad \text{with} \quad k = 1,$$

$$x \in D_2 = \{\xi_1(\tau) < x < \xi_2(\tau)\} \quad \text{with} \quad k = 2,$$

$$\tau > 0,\ \ \xi_1(+0) = \xi_0 > 0,\ \ \xi_2(\tau) = a\xi_1(\tau),\ \ a > 1,$$

$$t_1(x,0) = \left(1 - \frac{x}{\xi_0}\right) t_e, \tag{2}$$

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