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THE TRICOMI PROBLEM FOR AN EQUATION OF MIXED TYPE WITH A NONSMOOTH LINE OF POWER DEGENERACY

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1. Consider the equation

$$L(u) \equiv \operatorname{sgn} y|y|^n u_{xx} + x^m u_{yy} + \lambda \operatorname{sgn} y x^m |y|^n u = 0, \quad m, n > 0, \tag{1}$$

in a domain D bounded by a simple curve Γ, lying in the first quadrant, with the end points $A(1,0)$ and $B(0,b)$, $b > 0$, a segment OB of the axis Oy, and the characteristics OC and CA of equation (1) with $x > 0$, $y < 0$. Here $O(0,0)$, $C(x_C, y_C)$, $x_C = (1/2)^{1/\alpha}$, $y_C = -(1/2)^{1/\beta}$, $2\alpha = m+2$, $2\beta = n+2$, $p = n/2(n+2)$, $q = m/2(m+2)$.

In the domain D for equation (1) let us formulate the following problem.

The Tricomi problem (Problem T). Find a function $u(x,y)$ which satisfies the conditions

$$u(x,y) \in C(\overline{D}) \cap C^1(D) \cap C^2(D_+ \cup D_-); \tag{2}$$

$$Lu(x,y) \equiv g(x,y), \quad (x,y) \in D_+ \cup D_-; \tag{3}$$

$$u(x,y) = f(x,y), \quad (x,y) \in \Gamma; \tag{4}$$

$$u(0,y) = \varphi_1(y), \quad 0 \le y \le b; \tag{5}$$

$$u(x,y) = \psi(x,y), \quad (x,y) \in OC, \tag{6}$$

where $D_+ = D \cap \{y > 0\}$, $D_- = D \cap \{y < 0\}$, ψ, f, φ_1 are given sufficiently smooth functions, $f(0,b) = \varphi_1(b)$, $\varphi_1(0) = \psi(0,0)$.

Note that problem (2)–(6) for equation (1) with $n > m > 0$, $\lambda = 0$ is studied in [1]. In [2], one establishes the principle of maximum and on its base obtains the uniqueness theorem for a solution to Problem T for equation (1) with $\lambda = 0$ and $n > 0$, $m > -n/(n+1)$. In [3], one constructs a solution in the form of a biorthogonal series and proves the uniqueness of a solution of the boundary value problem for an equation of mixed type with one line of change of type by the method of integral identities. In [4], using the method of spectral analysis, one solves the Tricomi problem for equation (1) with $m = n > 0$ and $\lambda = 0$. This paper follows those [4]–[6]. Here in the class of regular in the domain D solutions to equation (1) we obtain the uniqueness theorem for a solution to problem (2)–(6) for all $\lambda \le 0$ and $n = m$, provided λ is a real parameter and for all

$$4\lambda_2^2 \le -\lambda_1(7(q-p)^2 + 2(p+q)), \quad \lambda_1 \le 0, \quad n > m, \tag{7}$$

if $\lambda = \lambda_1 + i\lambda_2$. In the case, when $n = m > 0$, $g(x,y) = \varphi_1(y) = \psi(x,y) = 0$ and the curve Γ coincides with the "normal" curve $\Gamma_0 : x^{2\alpha} + y^{2\alpha} = \alpha^2$, we construct a solution to the Tricomi problem for equation (1) on the base of the theory of series of eigenfunctions of the corresponding spectral problem.

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