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ONE DEFINITION OF ENTIRE FUNCTIONS OF EXPONENTIAL TYPE ON HOMOGENEOUS MANIFOLDS

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A Riemann manifold M is said to be two-point homogeneous (t. h.) if for any pairs of points $p_1, p_2 \in M$ and $q_1, q_2 \in M$, satisfying the condition $\rho(p_1,p_2) = \rho(q_1,q_2)$ ($\rho(p,q)$ is the distance between the points p and q), an isometry g of the manifold M exists such that $g(p_1) = q_1$ and $g(p_2) = q_2$. As is well-known (see [1]), all the t. h. manifolds are Euclidean spaces, the circle S^1, and symmetric spaces of rank 1 of compact and noncompact types. In recent years, various problems of the theory of approximation of functions on t. h. manifolds are actively studied (see, e. g., [2]–[5]). In the capacity of the apparatus of approximation on compact t. h. manifolds they use spherical polynomials, on noncompact t. h. manifolds in the capacity of approximating functions some analogs of entire functions of exponential form should be used. Thus in [5], [6], in the capacity of the approximation apparatus the entire vectors of exponential type in Banach spaces were used, but these vectors are not functions on the space M. In this article, in the spaces $L_p(M)$, we define entire L_p-functions of exponential type for an arbitrary t. h. manifold M. It is established that for the Euclidean space the introduced functions coincide with the ordinary entire functions of exponential type, which belong to the space L_p, while for compact t. h. manifolds the introduced functions coincide with the spherical polynomials.

We fix the permanent notation. We denote by M a t. h. Riemann manifold, G stands for the group of all isometries of the Riemann manifold M, o means a certain fixed point in M, K stands for the stationary subgroup of the point o in the group G. The group G intrinsically transitively acts on M and, respectively, the space M can be identified with the factor-space of left adjacent classes $M = G/K$; in this situation, the point o is identified with the adjacent class eK (e being the unit in the group G).

For any set X with measure $d\sigma$, as usual, we denote by $L_p(X, d\sigma)$ the Banach space (BS) consisting of measurable complex-valued functions $f(x)$ on X with the finite norm

$$\|f\|_p = \|f\|_{L_p(X)} = \left(\int_X |f(x)|^p \, d\sigma\right)^{1/p}, \quad 1 \le p < \infty.$$

If X is a metric space, then we denote by $L_\infty(X)$ the BS of uniformly continuous bounded functions on X with the norm

$$\|f\|_\infty = \|f\|_{L_\infty(X)} = \sup_{x\in X} |f(x)|.$$

$C(X)$ will stand for a set of all continuous functions on X, and $C_c(X)$ for a set of continuous functions with compact support (all functions are assumed to be complex-valued). In particular, on the Riemann manifold M the BS $L_p(M) = L_p(M, dx)$ and $L_\infty(M)$ arise, where dx is an element of the Riemann measure on M.

Let T_xM be a set of tangent vectors to the manifold M, while $S(x)$ a set of unit tangent vectors (unit sphere) at the point $x \in M$. Let B be the manifold of all unit tangent vectors to

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