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Spatial Equilibrium Problems for Auction Type Systems

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1. INTRODUCTION

At the present time rather a lot of spatial equilibrium models exist which are utilized for description of behavior and for management of complex systems containing many interacting elements, with taking into account their spatial distribution. Such models are greatly developed in economics where they usually describe systems of markets joined by transport communications within either perfect (in the sense of Walras) or imperfect (in the sense on Cournot–Bertrand) competition conditions (see, e.g., [1–3]). However, these kinds of competition are not sufficient for an adequate description of modern management mechanisms in economic systems. In particular, this is the case for systems related to natural monopolies in energy sectors where a state permits their privatization, but keeps certain tools for influence in this field, for instance, via the price policy (see, e.g., [4–6]). Since the optimal control of such a system requires consideration of both separate markets (participants) reactions and capacity and topology restrictions, the hierarchic (multilevel) optimization and game theory problems are usually utilized as suitable mathematical models. As a result, one must solve either very complicated global optimization problems having equilibrium constraints, or mixed integer programming problems. It is very difficult to find then efficient computational solution methods, especially for high-dimensional problems, which arise typically in applications.

Instead of the classical perfectly and imperfectly competitive markets one can utilize an auction based market model where the clearing price is established on current volume and price announcements of participants for buying and selling a homogeneous commodity in order to maintain the constant balance between demand and supply. In [7] (see also [3], Section 5.4), a variational inequality format for the homogeneous commodity auction market model was proposed. In [8], this approach was extended for markets with many commodities. In these models, auction prices are determined implicitly as Lagrange multipliers. As a result, rather simple and natural conditions of existence and uniqueness of solutions were obtained and also efficient numerical methods were proposed.

In this paper, we propose to apply the above approach to modelling of a system of auction markets joined by transmission lines subject to joint balance and capacity flows constraints which are typical for energy systems. As a result, we construct a single-level variational inequality model which involves implicitly a part of constraints. Taking this model as a basis, we propose iterative solution methods for the initial problem. We also show that under certain additional monotonicity conditions the model becomes equivalent either to a nonsmooth convex optimization problem or to a saddle point problem, where one of the functions is computed algorithmically with the help of independent solutions of separate auction problems.

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