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SOLVING NONLINEAR EQUATIONS IN ANALYTICAL POLYALGEBRAS. II

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In this part of the article we proceed with the investigation started in [1], where we developed a necessary algebraic formalism. Here we apply our method to the periodic Cauchy problems (see [2]) for evolutionary systems, and obtain existence theorems, among them results similar to Nash's theorem and Nishida's theorem (see [3], [4]) on the existence of a solution of complete nonlinear Navier–Stokes system (see [5]).

In [1] we exposed the investigation technique, the combinatorial algebraic calculus in polyalgebras, which unifies L.V. Ovsyannikov's approach (see [6]) and the Nishida–Treves approach (see [7]) and develops the ideas due to A.F. Sidorov (see [2]) and his disciples. Here we shall specify our method in order to prove new theorems on existence of periodic solutions of the evolutionary systems; in particular, on the existence of solutions of the Cauchy problem periodic with respect to the spatial variables for the complete Navier–Stokes system which describes the motion of a compressible viscous heat conducting gas.

Let us consider the equation

$$u_t = \mathcal{M}u + L_2[u,u] + L_3[u,u,u] + \cdots \tag{1}$$

in the pronilpotent polyalgebra A of exponential series (see [1]). In order to apply the theory developed in [1], we shall consider series which generalize the Fourier series (see [9], [10]) in a special way (see [8], [2]) adapted to nonlinear problems (see [2], [11]–[13]).

In equation (1) the function $u(x,t) \in \mathbf{C}^m$, where $x \in R^n$, $t \in [0,+\infty)$, is unknown, and $L_k[a_1,\dots,a_k]$ stands for a k-linear mapping, i. e., a polynomial of degree k in $a_1,\dots,a_k$ and their derivatives with respect to $x_1,\dots,x_n$ of order $\le P$ ($P>1$) whose coefficients are analytic periodic vector-functions of spatial arguments (or, for the sake of simplicity, are constants in $\mathbf{C}^m$).

Let us construct a solution $u(x,t)$, which is periodic in $x_1,\dots,x_n$, as follows (see [8], [2]):

$$u^j(x,t) = \sum_{\alpha_1=0}^{\infty}\cdots\sum_{\alpha_n=0}^{\infty}\sum_{\beta_1=0}^{\infty}\cdots\sum_{\beta_n=0}^{\infty} g^j_{\alpha_1\dots\alpha_n\beta_1\dots\beta_n}(t)\xi_1^{\alpha_1}\cdots\xi_n^{\alpha_n}\eta_1^{\beta_1}\cdots\eta_n^{\beta_n},$$

where $\xi_\nu = e^{ix_\nu}$, $\eta_\nu = e^{-ix_\nu}$ ($\nu=1,\dots,n$, $j=1,\dots,m$). Note that this series sometimes is called a normal S.N. Bernshteĭn series (see [14]). Then, in the case where the right-hand side of system identically vanishes with $u^j = c^j = u^j|_{\xi=\eta=0}$, where c^j are arbitrary constants, the coefficients of this series satisfy a recurrent sequence of ordinary differential equations. It is convenient to use the pronilpotent ideal $A = J_1$ and replace $u \mapsto v$, where $v^j = u^j - c^j$. Having denoted again by u the v, we then have $u^j|_{\xi=\eta=0} = 0$, and all the constants c^j now appear in the coefficients of equation (1).

For the multiindices $\alpha = (\alpha_1,\dots,\alpha_n)$, $\beta = (\beta_1,\dots,\beta_n)$, we set $N = |\alpha| + |\beta| = \alpha_1 + \cdots + \alpha_n + \beta_1 + \cdots + \beta_n$; then the space J_N/J_{N+1} (see [1]) is the direct sum of all m-dimensional subspaces

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