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ANALYSIS OF PROCESSES OF HEAT TRANSFER WITH REGARD FOR RELAXATION IN MEDIUM OF HEAT FLOW AND VOLUME SOURCES OF ENERGY

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The investigation of heat transfer by means of representation of the density of the heat flow W in the form of the product of the temperature gradient T and the heat conductivity index K_* (the Fourier law)

$$W = -K_* \operatorname{grad} T \tag{1}$$

is not always applicable. It is restricted by the requirement of the smallness of length and time of a free run of particles in comparison with characteristic spatial-time scales of the change of temperature.

Along with (1), the following hyperbolic heat conduction equation of the form

$$W = -K_* \operatorname{grad} T - \tau \frac{\partial W}{\partial t} \tag{2}$$

is known, where into the right side the derivative of W by time enters, τ is the time of relaxation of the density of the heat flow. With constant values of parameters K_* and τ model (2) widely and for a long time is used in many domains of the physics (see, e. g., [1]–[3]). In [4]–[6] (see also the bibliography therein) the analysis of the processes of heat transfer with regard for relaxation of heat flow is carried out in high-temperature media with participation of nonlinear dependence of coefficients K_* and τ on some thermodynamic quantities.

In this article the heat transfer by means of representation of heat flow in the form (2) is investigated with volume source or sinks in medium being taken into account.

One of distinguishing features of the description of heat transfer with the use of hyperbolic model (2) is the presence of discontinuous solutions. The temperature and density of flow on the front of the temperature wave possess a strong discontinuity.

The fact that volume sources or sinks of energy are taken into account brings a certain specificity to the character of distribution of the sought-for values.

1. Let us consider the heat transfer in a stationary medium in approximation to a plane symmetry. Let ρ be the density of the medium. We choose the variable $m = \rho x$ as the spatial coordinate. The corresponding equation of energy can be written as follows

$$C_v \frac{\partial T}{\partial t} = -\frac{\partial W}{\partial m} + Q, \tag{3}$$

where $C_v = \text{const}$ is the heat capacity of the medium, Q is either volume source ($Q > 0$), or sink ($Q < 0$) of energy. The change of the density of heat flow is described by the equation

$$W = -K \frac{\partial T}{\partial m} - \tau \frac{\partial W}{\partial t}, \quad K = K_* \rho. \tag{4}$$