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Phenomenologically Symmetric Geometry of Two Sets of Rank (3,2)

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Abstract—We define and study one of the simplest phenomenologically symmetric geometry of two sets of rank (3, 2), given on one- and two-dimensional manifolds by metric function.

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Consider one- and two-dimensional manifolds M and N, whose points we denote by lowercase Latin and Greek letters, respectively, and metric function f, which ascribes real number $f(i\alpha) \in R$ to some of the pairs $\langle i\alpha\rangle \in M \times N$. Note that metric function f ascribes a number to a pair of points from *different* sets, and thus it cannot be an ordinary metric. First of all, suppose that the following two axioms are satisfied.

A1. The domain of f is open and dense in $M \times N$.

A2. Metric function f is smooth enough.

Let x and ξ, η be local coordinates on manifolds M and N, respectively, then metric function f has coordinate representation

$$f = f(x, \xi, \eta), \tag{1}$$

and, for example, $f(i\alpha) = f(x_i, \xi_\alpha, \eta_\alpha)$.

We assume also that the following axiom holds true.

A3. Local coordinates x and ξ, η are essential in representation (1).

In a mathematical way, axiom **A3** means the following inequalities:

$$\partial f(x, \xi, \eta)/\partial x \neq 0, \ \ \partial(f(x_1, \xi, \eta), f(x_2, \xi, \eta))/\partial(\xi, \eta) \neq 0, \tag{2}$$

for $x_1 \neq x_2$. We write these conditions also for some points on the manifolds:

$$\partial f(x_i, \xi_\alpha, \eta_\alpha)/\partial x_i \neq 0, \ \ \partial(f(x_j, \xi_\alpha, \eta_\alpha), f(x_k, \xi_\alpha, \eta_\alpha))/\partial(\xi_\alpha, \eta_\alpha) \neq 0, \tag{2'}$$

for $j \neq k$.

We construct a function F by mapping a tuple $\langle ijk, \alpha\beta\rangle \in M^3 \times N^2$ to the point $(f(i\alpha), f(i\beta), f(j\alpha), f(j\beta), f(k\alpha), f(k\beta)) \in R^6$. Consider the following axiom.

A4. The range of F lies on the smooth non-degenerate hypersurface in R^6

$$\Phi(f(i\alpha), f(i\beta), f(j\alpha), f(j\beta), f(k\alpha), f(k\beta)) = 0. \tag{3}$$

Definition. We say that metric function f defines *phenomenologically symmetric geometry of two sets* (PS GTS) of rank (3, 2) on one- and two-dimensional manifolds M and N, if axioms **A1–A4** are satisfied.

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