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Certain Classes of Problems on Linear Conjugation for a Two-Dimensional Vector Admitting Explicit Solutions

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Abstract—We consider the structure of the set of piecewise meromorphic solutions to the linear conjugation problem for a two-dimensional vector and study its relations with the fractional-linear conjugation problem. We prove that under the assumption of the piecewise meromorphic solvability of any of these problems there exists a canonical system of solutions to the linear conjugation problem and describe classes of problems which are solvable in a closed form.

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Let Γ be a simple smooth closed contour dividing the complex plane into two domains: D^+ and D^- $(0 \in D^+, \infty \in D^-)$,

$$G(t) = \begin{pmatrix} g_{11}(t) & g_{12}(t) \\ g_{21}(t) & g_{22}(t) \end{pmatrix}, \quad \Delta(t) = \det G(t) \neq 0, \ \ t \in \Gamma, \tag{0.1}$$

and let H be a continuous on Γ matrix function of the second order. The homogeneous linear conjugation problem for a two-dimensional vector consists in the evaluation of a piecewise-holomorphic function $\mathbf{w}(z) = \big(w^1(z), w^2(z)\big)$ with H-continuous on Γ boundary values $\mathbf{w}^{\pm}(t)$ satisfying the condition

$$\mathbf{w}^+(t) = G(t)\mathbf{w}^-(t), \tag{0.2}$$

or, in the scalar form, the conditions

$$\begin{aligned} w^{1+}(t) &= g_{11}(t)w^{1-}(t) + g_{12}(t)w^{2-}(t), \\ w^{2+}(t) &= g_{21}(t)w^{1-}(t) + g_{22}(t)w^{2-}(t). \end{aligned} \tag{0.3}$$

See the monograph [1] (pp. 17–52) for the qualitative theory of problem (0.2) (of any dimension) in the Hölder classes of functions, and [2] for this theory in more extensive classes of matrix functions. But there are only few examples of matrix functions, for which the problem can be solved explicitly (in terms of the Cauchy integrals). One of such examples is considered in [3] in the problem with a triangular matrix function of the second order. The linear conjugation problem for meromorphic matrix functions of any order is studied in the monograph [2]. A constructive algorithm for solving the linear conjugation problem for meromorphic matrix functions of the second order is proposed in [4]. A special class of matrix functions of the second order reducible to triangular matrix functions (by left and right multiplication by rational matrices) is described in [5]. The study is continued in the paper [6] devoted to the classification of matrix functions of the second order (which allow the mentioned transforms) in dependence of the number of linearly independent elements of a matrix function over a field of rational functions. In this paper we establish certain conditions for elements of the matrix function (0.1) ensuring the explicit solvability of problem (0.3).

The paper consists of three parts. In the first Section we study the structure of the set of piecewise meromorphic solutions to problem (0.3). In the second Section we generalize results of the paper [7] and

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