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ANALYTIC CYCLES ON COMPLEX TORI

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The transcendence degree of the field of meromorphic functions on a compact complex manifold is one of the most important invariants related to the geometry of the manifold. For example, from the well-known classification of compact complex surfaces it follows that on a surface with one algebraically independent meromorphic function a pencil of elliptic curves exists. More exotic manifolds exist without nonconstant meromorphic functions, still scantily explored, which are usually used in the literature for constructing examples disproving the ordinary geometric intuition (see [1]).

This article is devoted to the study of the geometry of simple complex tori without nonconstant meromorphic functions, which allows us to get certain information concerning geometry of manifolds which are close to such tori in the sense of the theory of deformations of complex structures.

Theorem 1. *Let V be a connected smooth closed complex-analytic submanifold of a complex torus T. If $0 < \dim(V) < \dim(T)$, then V is unstable with respect to small deformations of T. Moreover, for the normal bundle N of V in T, we have $H^1(V, N) \neq (0)$.*

As is known (see [2], theorem 1), $H^1(V, N) = (0)$ implies stability of V. Let $X \to S$ be a Kuranishi versal family of deformations of a torus $T = X_0$, and $\Delta \subset S$ the set of points $s \in S$ such that the torus X_s is simple and on X_x no nonconstant meromorphic functions exist. It is well-known that Δ is dense in S.

Theorem 2. *On a simple complex torus T without nonconstant meromorphic functions, any analytic cycle of codimension not less than 1 is 0-dimensional.*

Proof. As is known (see [1]), T has no nonzero divisors. Show that no curves exist on T. Let $C \subset T$ be an irreducible curve. One can assume that $0 \in C$. Let $C_n = C + \dots + C$ (n summands). If $C_{n-1} \neq T$, then $\dim(C_n) = n$. Indeed, $C_{n-1} \subset C_n$ and $C_{n-1} \neq C_n$ (otherwise $C_{n-1} = C_{n-1} + C_{n-1}$ and, consequently, C_{n-1} is a nontrivial analytic subgroup in T, which contradicts the assumption that T is a simple torus), therefore $\dim(C_n) = 1 + \dim(C_{n-1})$ and then $\dim(C_n) = n$. If $g = \dim(T)$, then C_{g-1} is a divisor on T, which contradicts Krasnov's theorem (see [1]).

Let $S \subset T$ be an irreducible reduced closed complex subspace of minimal strictly positive dimension on T. Clearly, $\dim(S) > 1$, $\operatorname{Sing}(S)$ is finite, on S there are no nontrivial divisors and, therefore, no nonconstant meromorphic functions. Resolving (isolated) singularities of S with the use of successive monoidal transformations of T with nonsingular centers (see [3], Chap. 0, § 7, theorem I′), we obtain a smooth model S' of the space S such that $S' \subset T'$ where T' is obtained by successive blowing-up of nonsingular centers of the Kähler manifold T, and so T' is a Kähler manifold. Consequently, S' is a Kähler manifold without nonconstant meromorphic functions.