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OBSTRUCTION TO INTEGRABILITY OF INFINITESIMAL DEFORMATION OF k-DUAL STRUCTURE

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The aim of this article is to construct an obstruction to the integrability of infinitesimal deformation of the structure of a manifold over the algebra of k-dual numbers. We prove that, as in the case of complex structure, this obstruction is the differential bracket $[V, V]$ of an infinitesimal deformation V. This bracket determines a class in the second cohomology group with coefficients in the sheaf of holomorphic vector fields.

All the geometric objects are supposed to be smooth.

1. An infinitesimal deformation of k-dual structure

Let M be a compact smooth manifold of dimension $n(k+1)$, endowed with a k-dual structure $\overline{J}$. This structure is given by a collection of the affinor fields $\overline{J} = \{J_s : M \to T^1_1 M\}$, $s = \overline{1,k}$, such that at each point x in M the following conditions hold:

1) $J_s(x) \cdot J_r(x) = 0 \ \ \forall s, r = \overline{1,k}$;

2) $\operatorname{rank}\|J_s(x)\| = n$, $s = \overline{1,k}$, where $\|J_s(x)\|$ is a matrix of J_s;

3) on M an atlas exists

$$\mathcal{A} = \{(U_\alpha, \varphi_\alpha)\}, \ \text{ where } \ \varphi_\alpha : x \to (X^i = x^i_0 + \varepsilon_1 x^{n+i}_1 + \cdots + \varepsilon_k x^{n+i}_k), \ \ x^i_0, x^{n+i}_p \in \mathbb{R}, \ \ i = \overline{1,n},$$

such that the affinors $J_s(x)$ possess constant coordinates with respect to this atlas. Then, with respect to the basis $\{\frac{\partial}{\partial x^i_0} = \partial^0_i, \ \frac{\partial}{\partial^{n+i}_p} = \partial^p_{n+i}\}$, the matrices $\|J_s(x)\|$ of affinors J_s have the form

$$\|J_s(x)\| = \begin{pmatrix} 0^{ns}_n, & 0^{ns}_{nk} \\ E^n_n, & 0^n_{nk} \\ 0^{n(k-s)}_n, & 0^{n(k-s)}_{nk} \end{pmatrix},$$

where E^n_n is the identity $n \times n$-matrix, 0^β_α the zero $\alpha \times \beta$-matrix. $\|J_s(x)\|$ will be called canonical. Note that on a manifold M a k-dual structure exists if and only if M is a manifold over algebra of k-dual numbers $\mathbb{R}(\varepsilon_1, \ldots, \varepsilon_k) = \{a_0 + a_1\varepsilon_1 + \cdots + a_k\varepsilon_k \mid \varepsilon_s \cdot \varepsilon_p = 0, \ s, p = \overline{1,k}, \ a_i \in \mathbb{R}, \ i = \overline{0,k}\}$ (see [1]).

Let us consider the linear frame bundle $(L(M), \pi, M)$. The structure of manifold over algebra $\mathbb{R}(\varepsilon_1, \ldots, \varepsilon_k)$ determines a G-structure $P(M, G) \subset L(M)$ with the Lie subgroup $G = \{A \in GL(n(k+1)) \mid A\|J_s\| = \|J_s\|A, \ s = \overline{1,k}\}$, where $\|J_s\|$ are the canonical matrices of the structure affinors J_s.

For any Lie group G, a G-structure is said to be integrable if an adapted atlas exists such that the Jacobians of coordinate transformations are lying in G. An integrable G-structure determines a section $s : M \to E_G$ of the associated bundle $p : (E_G = L(M)/G) \to M$ (see [2]).