ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2009, Vol. 53, No. 6, pp. 7–14. © Allerton Press, Inc., 2009.
Original Russian Text © H. Kim, R.V. Namm, E.M. Vikhtenko, and G. Woo, 2009, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2009, No. 6, pp. 10–19.

Regularization in the Mosolov and Myasnikov Problem with Boundary Friction

H. Kim[1*], R. V. Namm[2], E. M. Vikhtenko[2***], and G. Woo[1****]**

[1]*College of Natural Sciences, Changwon National University, Changwon, 641-773, South Korea*
[2]*Pacific National University, ul. Tikhookeanskaya 136, Khabarovsk, 680035 Russia*

Received February 14, 2007

Abstract—We propose an iterative algorithm for solving a semicoercive nonsmooth variational inequality. The algorithm is based on the stepwise partial smoothing of the minimized functional and an iterative proximal regularization method.
We obtain a solution to the variational Mosolov and Myasnikov problem with boundary friction as a limit point of a sequence of solutions to stable auxiliary problems.

DOI: 10.3103/S1066369X09060024

Key words and phrases: *variational inequality, Mosolov and Myasnikov problem, functional, minimization, proximal regularization, finite element method.*

1. PROBLEM DEFINITION

Let $\Omega \subset R^2$ be a finite domain with a sufficiently smooth boundary Γ. Consider the problem

$$J(v) = \frac{1}{2}\int_\Omega |\nabla v|^2\, d\Omega - \int_\Omega f v\, d\Omega + \int_\Omega g_1 |\nabla v|\, d\Omega + \int_\Gamma g_2 |v|\, d\Gamma \to \min,\ \ v \in W_2^1(\Omega), \qquad (1)$$

where $g_1 > 0$, $g_2 > 0$, g_1, g_2 is a constant, $f \in L_2(\Omega)$.

The functional $J(v)$ is nondifferentiable and not strongly convex in the space $W_2^1(\Omega)$. In [1] one studies a partially smoothed problem in the form

$$J_\varepsilon(v) = \frac{1}{2}\int_\Omega |\nabla v|^2\, d\Omega - \int_\Omega f v\, d\Omega + \int_\Omega g_1 \sqrt{|\nabla v|^2 + \varepsilon^2}\, d\Omega + \int_\Gamma g_2 |v|\, d\Gamma \to \min,\ \ v \in W_2^1(\Omega), \quad (2)$$

where ε is a sufficiently small positive parameter.

In contrast to the classical Mosolov and Myasnikov problem with the adhesion boundary condition [2], the uniqueness theorem is not proved for problem (1), because the quadratic form $\int_\Omega |\nabla v|^2\, d\Omega$ is only a seminorm in the space $W_2^1(\Omega)$.

The uniqueness theorem for problem (2) in the space $W_2^2(\Omega)$ is proved in [1].

Problem (2) is equivalent to the variational inequality

$$u \in W_2^1(\Omega) : \int_\Omega \left(\nabla u \nabla (v-u) + g_1 \frac{\nabla u \nabla (v-u)}{\sqrt{|\nabla u|^2 + \varepsilon^2}} - f(v-u) \right) d\Omega$$
$$+ \int_\Gamma g_2 \left(|v| - |u|\right) d\Gamma \ge 0 \ \ \forall\, v \in W_2^1(\Omega).$$

*E-mail: dream7734@hanmail.net.
**E-mail: namm@mail.khstu.ru.
***E-mail: vikht@mail.khstu.ru.
****E-mail: gswoo@changwon.ac.kr.