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Index for Linear Systems of Differential-Algebraic Equations with Partial Derivatives

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Abstract—We investigate time-varying linear differential algebraic equations with partial derivatives. We introduce concept of insolubility index as the least possible order of the differential operator which transforms the initial system into a structural form with separated the "algebraic" and "differential" subsystems. The approach does not assume the existence of differential indexes with respect to independent variables.

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1. INTRODUCTION

We consider the system of equations

$$A(x,t)\frac{\partial u}{\partial x} + B(x,t)\frac{\partial u}{\partial t} + C(x,t)u = f(x,t), \tag{1.1}$$

where $u \equiv u(x,t)$ and $f(x,t)$ are the desired and given n-dimensional vector-functions, respectively, $A(x,t)$, $B(x,t)$, and $C(x,t)$ are given $n \times n$-matrices defined in the domain

$$\mathbb{U} = \{(x,t) : x \in X = [x_0, x_1],\ t \in T = [t_0, t_1]\} \subset \mathbb{R}^2.$$

We assume that coefficients and the right-hand side in (1.1) have in $\mathbb{U}$ partial derivatives of sufficiently large orders, and

$$\det A(x,t) = 0,\ \ \det B(x,t) = 0\ \ \forall (x,t) \in \mathbb{U}. \tag{1.2}$$

We allow cases of degeneration of the matrix $C(x,t)$ both on the whole set $\mathbb{U}$ and on some its subset.

Systems of the form (1.1) with condition (1.2) are called *systems of differential-algebraic equations (DAE) with partial derivatives*.

DAE with partial derivatives arise in modeling of various processes in hydrodynamics, physics of atmosphere, plasma physics, electronic schemes, nanoelectronics and other fields [1–5].

One of important characteristics of that systems (both for analysis and for numerical solution) is an index. It shows a measure of insolubility of the system with respect to derivatives. The conditions of existence of the index determine an approach to investigation of degenerated system of equations.

Certain attempts to introduce the concept of an index for DAE with partial derivatives have been made repeatedly (see, e.g., [6–9]), but so far we have not its universally accepted definition. This is valid also for not resolved with respect to derivatives systems of ordinary differential equations, which are called DAE,

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