Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 49, No. 5, pp.52–58, 2005 *UDC 514.752:512.544*

BOL THREE-WEBS GENERATED BY FOLIATIONS OF DIFFERENT DIMENSIONS

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In the classical theory of three-webs generated by foliations of *equal* dimensions, special classes of three-webs are singled out: Reidemeister webs, Thomsen webs, Bol webs, Moufang webs [1]. These classes are characterized, on the one hand, by the property that the corresponding configurations R, T, B, and M are closed, and from the other hand by the associativity identity, the commutativity identity, the Bol identity, and the Moufang identity, respectively, which hold in the coordinate loops of these webs.

The theory of webs generated by surfaces of different dimensions was developed in [2]. It turned out that it is impossible to generalize directly the above mentioned configurations and identities to a web generated by foliations of *different* dimensions since the coordinate groupoid determined by this web is not a quasigroup, in general. In [3], [4] we have generalized the associativity property and the configuration R to the three-web $W(p,q,p+q-1)$ generated by foliations of dimensions p, q, and $p+q-1$, and to the web $W(p,q,q)$ generated by foliations of dimensions p, q, and q. In the present paper, by analogy with the classical theory, we define the left Bol three-web $B_l(p,q,q)$ generated by foliations of dimensions p, q, and q on a differentiable manifold of dimension $p+q$. We find necessary and sufficient conditions for a three-web $W(p,q,q)$ be the web $B_l(p,q,q)$, write structure equations for $B_l(p,q,q)$, and solve the equivalence problem for this class of webs. Finally, we find a three-web $B_l(2,3,3)$ with a simplest nonzero curvature tensor.

1. The most general structure equations for an arbitrary three-web $W(p,q,q)$ are given in [5]:

$$\begin{aligned} d\underset{1}{\omega}^\alpha &= \underset{1}{\omega}^\beta \wedge \Theta^\alpha_\beta + \mu^\alpha_{u\beta}\underset{1}{\omega}^u \wedge \underset{3}{\omega}^\beta - \mu^\alpha_{\beta\gamma}\underset{1}{\omega}^\beta \wedge \underset{1}{\omega}^\gamma, \\ d\underset{1}{\omega}^u &= \underset{1}{\omega}^v \wedge \omega^u_v + \underset{1}{\omega}^\beta \wedge \omega^u_\beta, \\ d\underset{2}{\omega}^\alpha &= \underset{2}{\omega}^\beta \wedge \Theta^\alpha_\beta + \mu^\alpha_{\beta\gamma}\underset{2}{\omega}^\beta \wedge \underset{2}{\omega}^\gamma, \end{aligned} \tag{1}$$

where $\alpha,\beta,\gamma,\ldots = 1,2,\ldots,p$; $u,v,w,\ldots = p+1,p+2,\ldots,q$, $\mu^\alpha_{u\beta}$ and $\mu^\alpha_{\beta\gamma}$ are skewsymmetric with respect to lower indices and form the torsion tensor for $W(p,q,q)$ [2]. Then the foliation λ_1 is given by the equations $\underset{1}{\omega}^\alpha = 0$, $\underset{1}{\omega}^u = 0$, the foliation λ_2 is given by $\underset{2}{\omega}^\alpha = 0$, and the foliation λ_3 by $\underset{3}{\omega}^\alpha \equiv \underset{1}{\omega}^\alpha + \underset{2}{\omega}^\alpha = 0$.

Let us sum up the first and the second equations in (1), and set

$$\Theta^\alpha_\beta = \omega^\alpha_\beta + \frac{1}{2}\mu^\alpha_{u\beta}\underset{1}{\omega}^u + \mu^\alpha_{\beta\gamma}(-\underset{2}{\omega}^\gamma + \underset{3}{\omega}^\gamma).$$

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