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n-ELEMENTARY EMBEDDABILITY AND n-CONDITIONAL TERMS

A.G. Pinus

In papers by the author [1], [2] the concept of conditional term was introduced and the relation between categories of embeddability of universal classes of algebras and conditional rational equivalence of these classes (by equal expressive possibilities defined on the algebras of these classes by means of terms) was established. In the present article the results obtain earlier will be expanded to the case where conditions are defined in a more general sense.

Let n be an arbitrary natural number or $n = e$, where $e \notin \omega$. An elementary formula of a certain signature σ, which has in the prenex normal form at most n changes of quantifiers, will be called n-formula. If $n = e$, then by n-formula we shall mean any elementary formula. By a complete set of n-conditions in the signature σ we shall call any set $\{\phi_1(\overline{x}), \dots, \phi_k(\overline{x})\}$ of n-formulas of this signature with free variables $\overline{x}$ such that the formula $\forall \overline{x} \bigvee\limits_{i=1}^{k} \phi_i(\overline{x})$ is identically true, and for any distinct $l, m \leq k$ the formulas $\phi_l(\overline{x}) \,\&\, \phi_m(\overline{x})$ are not executable.

Definition 1. We shall introduce the notion of an n-conditional term, where $n \in \omega$ or $n = e$, of signature σ by means of the functional differential induction:

a) all variables are n-conditional terms;
b) if $t_1(\overline{x}), \dots, t_k(\overline{x})$ are conditional terms of signature σ and $f(x_1, \dots, x_k)$ is the functional sign of this signature, then $f(t_1(\overline{x}), \dots, t_k(\overline{x}))$ is also an n-conditional term of this signature;
c) if $t_1(\overline{x}), \dots, t_k(\overline{x})$ are n-conditional terms, and $\phi_1(\overline{x}), \dots, \phi_k(\overline{x})$ is the complete set of n-conditions of signature σ, then

$$t(\overline{x}) = \begin{cases} \phi_1(\overline{x}) \to t_1(\overline{x}), \\ \dots\dots\dots \\ \phi_k(\overline{x}) \to t_k(\overline{x}) \end{cases}$$

is also an n-conditional term of this signature;
d) any n-conditional term is constructed for a finite number of steps by the rules a), b), c).

If $\mathfrak{A}$ is a universal algebra of signature σ, then any n-conditional term $t(\overline{x})$ ($n \in \omega$ or $n = e$) of given signature defines in a natural way a certain function (n-conditionally-termal function) on the main set of the algebra $\mathfrak{A}$.

Let us indicate the necessary interpretation only for the case c) from Definition 1: if $\overline{b}, a \in \mathfrak{A}$ and n-conditional term $t(\overline{x})$ is constructed of the n-conditional terms $t_1(\overline{x}), \dots, t_k(\overline{x})$ and the n-conditions $\phi_1(\overline{x}), \dots, \phi_k(\overline{x})$ with the help of the rule c), then $\mathfrak{A} \models t(\overline{b}) = a$ if and only if $\mathfrak{A} \models \phi_l(\overline{b})$ and $\mathfrak{A} \models t_l(\overline{b}) = a$ for a certain $l \leq k$.

One can easily see that the notion of a conditional term introduced in [1] corresponds to the notion of the 0-conditional term defined above.

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