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Boundedness and Compactness of Multidimensional Integral Operators with Homogeneous Kernels

O. G. Avsyankin* **and F. G. Peretyat'kin****
(Submitted by S.G. Samko)

Southern Federal University, ul. Mil'chakova 8a, Rostov-on-Don, 344090 Russia
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Abstract—We obtain sufficient conditions for the boundedness and compactness of multidimensional integral operators with homogeneous kernels acting from a weighted L_p-space to a weighted L_q-space.

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Introduction. Many works deal with multidimensional integral operators with homogeneous kernels acting in L_p-spaces (e.g., [1–7] and references therein). For such operators, boundedness conditions are known, as well as invertibility criteria and those for the Noetherian property; Banach algebras generated by such operators are described, and conditions of the applicability of the projection method are established. However, the action of operators with homogeneous kernels from L_p into L_q was not studied earlier.

Consider the operator

$$(K\varphi)(x) = \int_{\mathbb{R}^n} k(x,y)\varphi(y)\,dy, \quad x \in \mathbb{R}^n, \tag{1}$$

where the function $k(x,y)$ is defined on $\mathbb{R}^n \times \mathbb{R}^n$, measurable, and homogeneous of degree $(-n)$, i.e.,

$$k(\lambda x, \lambda y) = \lambda^{-n} k(x,y) \ \ \forall\, \lambda > 0.$$

In what follows we obtain sufficient conditions for the boundedness of K, which represents an operator from the space L_p with a power weight into L_q with another power weight. In addition, we prove certain sufficient conditions for the compactness of the operator $a(x)K$ under the assumption that the factor $a(x)$ belongs to a certain rather wide class of functions.

Below we use the following denotations: $\mathbb{R}^n$ is the n-dimensional Euclidean space, $x = (x_1, \dots, x_n) \in \mathbb{R}^n$, $|x| = \sqrt{x_1^2 + \cdots + x_n^2}$, $e_1 = (1, 0, \dots, 0)$, and $S_{n-1} = \{x \in \mathbb{R}^n : |x| = 1\}$.

1. Consider an operator K in the form (1). Theorem 4 in [8] implies that the operator K cannot act boundedly from the space $L_p(\mathbb{R}^n)$ into $L_q(\mathbb{R}^n)$ for $q \neq p$. In other words, if K is a bounded operator from $L_p(\mathbb{R}^n)$ into $L_q(\mathbb{R}^n)$, then $q = p$. Then it is natural to pass to the study of weighted L_p-spaces.

Let $1 \leqslant p < \infty$ and $\alpha \in \mathbb{R}$. We denote by $L_{p,\alpha}(\mathbb{R}^n)$ the space of all measurable on $\mathbb{R}^n$ complex-valued functions with the norm

$$\|f\|_{p,\alpha} = \left(\int_{\mathbb{R}^n} |f(x)|^p |x|^\alpha \, dx \right)^{1/p}.$$

*E-mail: avsyanki@math.rsu.ru.
**E-mail: pfg05@mail.ru.