ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2012, Vol. 56, No. 8, pp. 1–10. © Allerton Press, Inc., 2012.
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# On the Geometry of Point Correspondences of Three Conformal Spaces

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Received June 27, 2011

**Abstract**—Point correspondences of three conformal spaces are studied on the base of G. F. Laptev's invariant methods. We establish the basic equations and geometrical objects of the point correspondences in question. We construct invariant normalizations of the spaces, single out the basic tensors of the correspondences, establish a connection of the correspondences with the theory of multidimensional 3-webs, and find the torsion and the curvature tensors of a point correspondence. For a series of particular cases, we prove existence theorems.

**DOI:** 10.3103/S1066369X12080014

Keywords and phrases: *point correspondence, invariant normalization, torsion tensor, curvature tensor, multidimensional 3-web.*

1. *Basic equations.* For $n > 1$, consider three $n$-dimensional conformal spaces $\underset{\xi}{C}_n$ $(\xi, \eta, \theta = 1, 2, 3)$ with polyspherical coordinate systems [2]. Let $C : \underset{1}{C}_n \times \underset{2}{C}_n \to \underset{3}{C}_n$ be a point correspondence between these conformal spaces. Denote by $\underset{\xi}{M}_0$ corresponding points of the spaces $\underset{\xi}{C}_n$.

The correspondence $C$ generates six point mappings $\underset{\xi\eta}{T} : \underset{\xi}{C}_n \to \underset{\eta}{C}_n$ which arise when points $\underset{\theta}{M}_0$ of the spaces $\underset{\theta}{C}_n$, $\theta \neq \xi, \eta$, are fixed. We assume that, for fixed points of the spaces $\underset{\theta}{C}_n$, the mappings $\underset{\xi\eta}{T} : \underset{\xi}{C}_n \to \underset{\eta}{C}_n$ are regular and invertible on the pairs of spaces $\underset{\xi}{C}_n$, $\underset{\eta}{C}_n$.

To each point $\underset{\xi}{M}_0$ of $\underset{\xi}{C}_n$ we assign a conformal frame (see, e.g., [2]) formed by two points $\underset{\xi}{M}_0$, $\underset{\xi}{M}_{n+1}$ and $n$ linearly independent hyperspheres $\underset{\xi}{M}_i$ $(i, j, k, \ldots = 1, 2, \ldots, n)$ incident to the points $\underset{\xi}{M}_0$, $\underset{\xi}{M}_{n+1}$. The points and the hyperspheres of the frame satisfy the relations

$$(\underset{\xi}{M}_0, \underset{\xi}{M}_0) = 0, \ \ (\underset{\xi}{M}_{n+1}, \underset{\xi}{M}_{n+1}) = 0, \ \ (\underset{\xi}{M}_0, \underset{\xi}{M}_i) = 0, \ \ (\underset{\xi}{M}_i, \underset{\xi}{M}_{n+1}) = 0, \tag{1}$$

where the brackets denote the scalar products of elements defined by the basic quadratic form of the conformal space $\underset{\xi}{C}_n$.

Normalizing the points $\underset{\xi}{M}_{n+1}$, we let

$$(\underset{\xi}{M}_0, \underset{\xi}{M}_{n+1}) = -1. \tag{2}$$

Introduce the notation

$$(\underset{\xi}{M}_i, \underset{\xi}{M}_j) = \underset{\xi}{g}_{ij}. \tag{3}$$

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