Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 46, No. 3, pp. 47–55, 2002 *UDC 512.532*

A NEW CLASS OF DIVISORS OF SEMIGROUPS OF ISOTONE MAPPINGS OF FINITE CHAINS

V.H. Fernandes

Introduction

In 1987, at International Conference on semigroup theory in Szeged, J.-E. Pin posed the problem of *effective* description of a pseudovariety of semigroups $\boldsymbol{O}$ generated by all semigroups of isotone everywhere defined transforms of finite chains. In other words it is the search for an algorithm which would determine whether a given finite semigroup belongs to this pseudovariety. An observable progress in this problem started only since 1995. First P.M. Higgins (see [1]) proved that the pseudovariety $\boldsymbol{O}$ is selfdual and contains not all $\mathcal{R}$-trivial semigroups (thus, $\boldsymbol{O}$ is strictly less than $\boldsymbol{A}$, the pseudovariety of all finite aperiodic semigroups), though every finite band belongs to $\boldsymbol{O}$. The last result by Higgins was generalized in [2], where it was proved that every finite semigroup whose idempotents form an ideal, lies in $\boldsymbol{O}$. In [3] it was proved that the pseudovariety $\boldsymbol{POI}$, generated by all semigroups of injective isotone partial transforms of finite chains is a (proper) subpseudovariety in $\boldsymbol{O}$. On the other hand, in [4] it was shown that the interval $[\boldsymbol{O}, \boldsymbol{A}]$ of the lattice of all semigroup pseudovarieties possesses the continuum cardinality, and recently in [5] it was established that $\boldsymbol{O}$ is not finitely basable and, what is more, any semigroup pseudovariety $\boldsymbol{V}$ such that $\boldsymbol{POI} \subseteq \boldsymbol{V} \subseteq \boldsymbol{O} \vee \boldsymbol{R} \vee \boldsymbol{L}$, where $\boldsymbol{R}$ and $\boldsymbol{L}$ are the pseudovarieties of all finite $\mathcal{R}$-trivial semigroups and all finite $\mathcal{L}$-trivial semigroups, respectively, is not finitely basable. Nevertheless, the initial Pin problem remains unsolved.

The objective of this article is another additional movement towards the solution of the Pin problem. Namely, we will prove that $\boldsymbol{O}$ contains all semidirect products of chains (which are treated as semilattices) on semigroups of all injective isotone partial transforms of finite chains.

1. Preliminaries

We will use the standard notions and notation of the semigroup theory in [6]. However, in this article it seems to be convenient to denote by S^0 the *semigroup resulting of a given semigroup S by adjoining an additional zero* but not the *semigroup obtained from S by adjoining the zero if it fails to exist in S* (as was defined in [6]), i. e., under the assumption that 0 is a symbol not belonging to S, S^0 being the semigroup with the support $S \cup \{0\}$ and multiplication continuing the multiplication in S by the rule $0s = s0 = 00 = 0$ for all $s \in S$.

Let X be a set. We denote by $\mathcal{PT}(X)$ a monoid of all partial transforms of X (with respect to the composition), by $\mathcal{T}(X)$ a submonoid in $\mathcal{PT}(X)$, consisting of all everywhere defined transforms of X, and by $\mathcal{I}(X)$ a symmetric inverse semigroup on X, i. e., a submonoid in $\mathcal{PT}(X)$, consisting of all injective partial transforms of X. Consider an n-element chain X_n, for example, $X_n =$

The work was carried out within the frameworks of the project PRAXIS/2/2.1/MAT/73/94 of Centro de Algebra da Unversidade de Lisboa. Partially supported by the grant INVOTAN 9/A/96/PO.