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The Total Preservation of Unique Global Solvability of the First Kind Operator Equation With Additional Controlled Nonlinearity

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Abstract—For the Cauchy problem associated with an evolutionary operator equation of the first kind with an additional controlled term which nonlinearly depends on the phase variable, in a Banach space, we establish conditions for the total (on the set of admissible controls) preservation of unique global solvability under variation of the control parameter. We also establish the uniform bound for solutions. As examples, we consider initial-boundary value problems that are associated with a pseudoparabolic equation and a system of Oskolkov equations.

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INTRODUCTION

The total preservation of global solvability is the ability of a controlled system (with distributed or concentrated parameters) to preserve the global solvability with all admissible controls. For non-evolutionary systems, the notion of the *total preservation of solvability* is introduced analogously.

The term "total preservation of global solvability" was introduced in the paper [1]. Note that the violation of global solvability of an evolutionary controlled system associated with a differential or integro-differential equation is quite possible, if the growth rate of the right-hand side of the corresponding equation with respect to the phase variable exceeds the linear one (see the bibliography in [2] and in [3], Introduction, item 2). The presence of nonlinearity in the differential operator makes this situation even worse (e.g., [4]). See [2] for a detailed survey of papers devoted to the actual problem on criteria of the total preservation of global solvability and the background of this topic. Here we consider only the following two aspects.

1) For studying various control problems (along with the immediate postulation), many mathematicians use, as a rule, certain general results on sufficient conditions for the unique global solvability of nonlinear equations, in particular, those with fixed (independent of the control) right-hand side which nonlinearly depends on the state variable. When speaking about the general results, we mean that one, in fact, neglects the dependence of the nonlinearity on a certain collection of varying parameters (treated as a control). At the same time, it is possible that the global solvability (or its absence) essentially depends on the ranges of these parameters. In many cases, one can prove that if, for example, a system is globally solvable for a certain fixed control, then it keeps this property for all small (in a sense) variations of this control (while no general sufficient conditions for global solvability are fulfilled). This property is called the stable existence of global solutions (or, more generally, the preservation of global solvability), see, for example, the survey in [5]. In our case, the total preservation of unique global solvability is proved only under the condition of solvability of a certain majorant equation. This equation, in turn, is defined by a certain function $\mathcal{N}_1(\cdot)$ *which is one and the same* for all admissible controls. This is just the reason for the fact that the solvability of *only one* majorant equation guarantees the unique global solvability

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