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RECOVERY OF A FUNCTION WITH A FINITE NUMBER OF DISCONTINUITIES OF THE FIRST KIND VIA NOISY DATA

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The problem of the recovery of a function via noisy data is related, as a rule, to ill-posed problems. It was investigated by many authors in various spaces (see, e. g., [1], p. 192). In this article we consider the problem of recovery of an unknown function x by a given function x_δ; it is assumed that $x, x_\delta \in L_2(-\infty, +\infty)$, $\|x - x_\delta\|_{L_2} \leq \delta$, the level of error δ is known. In addition, we assume that for the desired function the following condition is a priori fulfilled:

Condition $(*)$. The function x has l discontinuities of the first kind and is continuously differentiable outside the discontinuity points. Both the function x and its derivative x' have at each discontinuity point both the left and right finite limits and are square integrable on the whole number axis. We will denote by s_i and Δ_i ($\Delta_i \neq 0$), $i = 1, 2, \dots, l$ the discontinuity points and the values of jumps, respectively. The following numbers $q \leq \min\{|\Delta_i|,\ i = 1, 2, \dots, l\}$ and $M \geq \max\{|s_i|,\ i = 1, 2, \dots, l\}$, are assumed to be known, while the number of discontinuities is unknown.

In this article we suggest a method which allows us to find the quantity of discontinuity points, determine approximately positions of discontinuities and values of jumps along with an estimation of the accuracy of their calculation. We construct an approximation of the initial function x outside the neighborhoods of discontinuity points, give a uniform estimate of the closeness of the approximation constructed to the desired function on the set $W_\delta \subset R \setminus \cup_{i=1}^{l}\{s_i\}$, which possesses the property $W_\delta \to R \setminus \cup_{i=1}^{l}\{s_i\}$ as $\delta \to 0$.

In the final part we discuss the question concerning the ways to carry over the results of the article to the situation where the function x is the exact solution of an integral equation of the first kind of convolution type.

1. Auxiliary assertions

By the function x, which satisfies condition $(*)$, we construct a function possessing the Gibbs effects of each discontinuity on non-intersecting intervals. The determination of approximations to discontinuity positions will be reduced to determination of maximums of this auxiliary function.

Let us investigate the case where $\delta = 0$. We define the direct and inverse Fourier transformations of the function x by the formulas

$$\widehat{x}(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(s) \exp(-izs)\, ds, \quad x(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{x}(z) \exp(izs)\, dz.$$

Put

$$x^B(s) = \frac{1}{\sqrt{2\pi}} \int_{-B}^{B} \widehat{x}(z) \exp(izs)\, dz, \quad B > 0.$$

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