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Riemann Boundary-Value Problem for Holomorphic Matrices on Non-Rectifiable Curve

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Abstract—We study the Riemann boundary-value problem on non-rectifiable curves for holomorphic matrices with Fokas–Its–Kitaev asymptotics by means of the Cauchy transforms of certain distributions with supports on that curves. The main results concern the existence of solutions of sufficiently large degree.

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INTRODUCTION

We consider a simple non-rectifiable arc Γ on the complex plane $\mathbb{C}$ beginning at point z_1 and ending at point z_2. The matrix Riemann boundary-value problem on this arc is a problem on reconstruction of holomorphic in $\mathbb{C} \setminus \Gamma$ matrix

$$Y(z) = \begin{pmatrix} Y_{11}(z) & Y_{12}(z) \\ Y_{21}(z) & Y_{22}(z) \end{pmatrix},$$

satisfying boundary condition

$$Y^+(t) = Y^-(t)G(t), \quad t \in \Gamma_0 := \Gamma \setminus \{z_1, z_2\}, \tag{1}$$

where $G(t)$ stands for defined on Γ matrix

$$G(t) = \begin{pmatrix} G_{11}(t) & G_{12}(t) \\ G_{21}(t) & G_{22}(t) \end{pmatrix},$$

and certain restrictions on behavior of desired matrix-function near the point at infinity and points $z_{1,2}$.

In classical case of piecewise-smooth (or at least rectifiable) curve Γ the main difficulties here are related with factorization of coefficient $G(t)$ (see, e.g, [1, 2]). If the factorization is found, then problem (1) is reduced to evaluation of few integrals of Cauchy type over path Γ. But for non-rectifiable paths that integral is undefined. In the present paper we overcome this obstacle. In this connection we solve problem (1) under assumption $G_{21}(t) \equiv 0$, which eliminates the factorization problem. Thus, we put

$$G(t) = \begin{pmatrix} G_{11}(t) & G_{12}(t) \\ 0 & G_{22}(t) \end{pmatrix}. \tag{2}$$

In addition, namely the Riemann boundary-value problem with triangular matrix-coefficient coefficients arise in the works, where matrix Riemann problem is applied for estimation of orthogonal polynomials

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