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INVESTIGATION OF THE CONVERGENCE OF ITERATION METHODS FOR SOLVING NONLINEAR PROBLEMS OF THE THEORY OF FILTRATION

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In the present article we construct an iteration process of the Udzava type for solving stationary nonlinear problems of the theory of filtration of an incompressible liquid, which obeys a discontinuous law of filtration with a limit gradient of displacement. Mathematically, the problem of filtration is formulated as a problem on the search for a minimum of a convex functional (with respect to the pressure) and a problem dual to the first one (with respect to a field of filtration velocities). The resolvability of these problems was investigated in [1]–[3]. Both the problems under consideration can be reduced to a problem of determination of a saddle point for a modified Lagrange function (see [4]). It is proposed to seek the saddle point of the modified Lagrange function by means of an algorithm of Udzava type. In [2] the convergence of algorithms of Udzava type was proved in the case of a strongly convex functional obtained from the initial one by a regularization, i. e., by the replacement of the discontinuous law of filtration with one close and continuous. In [5], an investigation of the convergence of analogous iteration methods for filtration problems with a discontinuous law (but, in fact, without limit gradient) was carried out. We prove the weak convergence of iteration approximations to the solutions of both initial and dual problems, as well as the strong convergence of discrepancies.

We shall write the filtration law in the form

$$v(u) = -g(|\nabla u|^2)\nabla u, \tag{1}$$

where v is filtration velocity, u is the pressure, $\xi \to g(\xi^2)\xi$ is a function defining the filtration law. With respect to this function we assume that the following conditions are fulfilled:

$$g(\xi^2)\xi = g_0(\xi^2)\xi + g_1(\xi^2)\xi, \tag{2}$$

$$g_0(\xi^2)\xi = 0 \quad \text{for} \quad \xi \le \beta, \tag{3}$$

$$\text{the function} \quad \xi \to g_0(\xi^2)\xi \quad \text{is absolutely continuous}, \tag{4}$$

both positive c_1 and c_2 exist such that with $\xi \ge \beta$ we have

$$c_1(\xi - \beta) \le g_0(\xi^2)\xi \le c_2(\xi - \beta), \tag{5}$$

a positive c_3 exists such that

$$0 \le (g_0(\xi^2)\xi)'_\xi \le c_3 \quad \text{for} \quad \xi \ge \beta, \tag{6}$$

$$g_1(\xi^2)\xi = \begin{cases} 0, & \xi \le \beta; \\ \vartheta, & \xi > \beta. \end{cases} \tag{7}$$

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