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Solvability of Some Boundary-Value Problems for Polyharmonic Equation with Hadamard–Marchaud Boundary Operator

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Abstract—We investigate solvability conditions for certain non-classical boundary-value problems for the polyharmonic equation. As the boundary operators we consider fractional differential operators in the sense of Hadamard–Marchaud. The considered problems generalize well-known Dirichlet and von Neumann boundary-value problems for boundary operators of fractional type.

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1. INTRODUCTION

Let $\Omega = \{x \in R^n : |x| < 1\}$ be n-dimensional unit ball, $n \geq 2$, and $\partial\Omega = \{x \in \mathbb{R}^n : |x| = 1\}$ be the unit sphere. Let $u(x)$ be polyharmonic (m-harmonic) function in domain Ω, α, μ are nonnegative real values, $r = |x|$, $r\frac{\partial}{\partial r} = \sum\limits_{\partial u}^{n} x_i \frac{\partial}{\partial x_i}$.

I. I. Bavrin [1] studied operators

$$\delta_\mu = r\frac{\partial}{\partial r} + \mu, \quad \delta_\mu^\ell = \left(r\frac{\partial}{\partial r} + \mu\right)^\ell, \quad \ell \in \mathbb{N},$$

$$\delta_\mu^{-1}[u](x) = \int_0^1 t^{\mu-1} u(tx)\, dt, \quad \delta_\mu^{(-\ell)}[u](x) = \int_0^1 t_1^{\mu-1} \cdots \int_0^1 t_\ell^{\mu-1} u(t_1 \dots t_\ell x)\, dt_\ell \dots dt_1.$$

One can show easily that

$$\delta_{\mu_1}^{(-\ell)}[u](x) = \frac{1}{(\ell-1)!} \int_0^1 t^{\mu-1} \left(\ln\frac{1}{t}\right)^{\ell-1} u(tx)\, dt.$$

As proved in [1], the operators $\delta_\mu^{(-\ell)}$ and δ_μ^ℓ are mutually inverse in the class of harmonic functions in the ball Ω. In the same paper these operators are applied for study of solvability of boundary-value problems for Laplace equation with boundary operator δ_μ^ℓ. We see from construction of the operator $\delta_\mu^{(-\ell)}$ that in the case $\mu > 0$ it is defined for noninteger meanings of ℓ, too. In this connection there arises a question on determination of inverse operator for $\delta_\mu^{(-\ell)}$ in the case of fractional ℓ.

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