ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2007, Vol. 51, No. 12, pp. 84–87. © Allerton Press, Inc., 2007.
Original Russian Text © E.A. Suleimanova, 2007, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2007, No. 12, pp. 83–86.

On the Properties of Holomorphically 2-Geodesic Transformations of the First Linear Type of Almost Hermitian Structures

E. A. Suleimanova[1]

[1] *Moscow State Pedagogical University, ul. Malaya Pirogovskaya 1, Moscow, 119992 Russia*

Received May 8, 2007; brief communication December 10, 2007

DOI: 10.3103/S1066369X07120079

The theory of geodesic mappings of pseudo-Riemannian spaces is one of the basic lines of investigation in the Riemannian geometry. p-geodesic mappings were studied by N. Sinyukov [1], S. Leiko [2] and others. In [3], p-geodesic curves and p-geodesic mappings of torsion-free affinely connected spaces were defined and p-geodesic mappings of linear and quadratic types were singled out.

In this paper, using the results of the above indicated research works, we study holomorphically 2-geodesic transformations of the first linear type of almost Hermitian structures. We introduce the notions of semispecial and special holomorphically 2-geodesic transformations and study properties of such transformations. We show that holomorphically projective and holomorphically geodesic transformations are particular cases of special 2-geodesic transformations. In terms of the introduced terminology, we prove that an almost Hermitian manifold admits no nontrivial holomorphically geodesic transformations.

Let (M, g) be a pseudo-Riemannian manifold, ∇ the Riemannian connection of the metric g, $\mathfrak{X}(M)$ the module of smooth vector fields on M.

In this paper, we use the following definitions.

A curve $\gamma : I \to M$ is called a *geodesic* ([4], P. 267) if the vector field

$$\dot{\gamma} = \left\{ \dot{\gamma}_s = \frac{d}{dt}\Big|_{t=s} \gamma(t),\ s \in I \right\}$$

is parallel along γ.

A curve $\gamma : I \to M$ is called a 2-*geodesic* ([2], P. 80–83) if, along it, there exists a 2-dimensional field of planes $E_2(t)$ which is parallel along γ with respect to ∇ and contains the tangent vector field of the curve, i.e., $\dot{\gamma}(t) \in E_2(t)$.

A diffeomorphism $\rho(2) : M \to M$ is called a 2-*geodesic transformation* ([3], P. 46) if, for each geodesic γ, its image $\overline{\gamma} = \rho(2) \circ \gamma$ is a 2-geodesic. In particular, if $E_2(t) = L(\dot{\gamma}, K(\dot{\gamma}))$, where K is an affinor, then $\rho(2)$ is called a 2-*geodesic transformation of the first linear type*.

An *almost Hermitian* (briefly, *AH*-) *structure* on M ([4], P. 320) is a pair $(J, g = \langle \cdot, \cdot \rangle)$, where J is an almost complex structure on M ([4], P. 300), g is a (pseudo-) Riemannian metric on M, and $\langle JX, JY \rangle = \langle X, Y \rangle$, $X, Y \in \mathfrak{X}(M)$. The endomorphism J is called the *structure endomorphism*. A manifold with fixed AH-structure is called an *almost Hermitian manifold*.

Let M^{2n} $(n \geq 2)$ be a smooth manifold with AH-structure (J, g). Unless otherwise stated, in what follows we always assume that $g = \langle \cdot, \cdot \rangle$ is a Riemannian metric.

Definition 1. A 2-geodesic transformation $g \to \widetilde{g}$ of the metric g of an AH-structure (J, g) is called a *holomorphically* 2-*geodesic transformation* if $(J, \widetilde{g})$ is also an AH-structure.

Definition 2. A holomorphically 2-geodesic transformation is called *trivial* if $\nabla = \widetilde{\nabla}$.