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Pierce Sheaf for Semirings with Involution

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Abstract—We introduce the concept of Pierce sheaf for semirings with involution, an analog of Pierce sheaf for rings. We construct maximal spectrum, Pierce congruence, Pierce sheaf of semirings with involution, Pierce stalk of semiring with involution. We prove main theorem on the isomorphism of semiring with involution and semiring with involution of global sections of Pierce sheaf.

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The fundamental work [1] by R. S. Pierce contains construction of ring sheaves on the Stone space of a ring which plays the role of basic space. These sheaves are named Pierce sheaves.

Any ring is isomorphic to the section ring of its Pierce sheaf. This structure is widely applied in the study of rings with large number of central idempotents. These rings possess nontrivial Pierce sheaf [2].

Later the Pierce sheaf construction was extended to another objects, for example, submodules [3], semirings [4], division rings [5].

Here we study the construction of the Pierce sheaf for semirings with involution and isomorphic representation of the involutive semiring as the Pierce sheaf sections.

1. $*$-IDEMPOTENTS, RING BS^*

Definition 1. A nonempty set S with binary operations $+$ and $\cdot$ is said to be a *semiring* if the following conditions hold:

1) $(S,+)$ is a commutative semigroup with the identity element 0,

2) $(S,\cdot)$ is a semigroup with unit,

3) the multiplication is distributive over addition in any possible operations combination,

4) $0a = 0 = a0$ for any $a \in S$.

Definition 2. A semiring S is called *a semiring with involution* if it possesses the unary operation $*$ such that the following conditions hold true:

1) $(a+b)^* = a^* + b^*$,

2) $(a^*)^* = a$,

3) $(ab)^* = b^*a^*$

for all $a, b \in S$.

For the sake of brevity we introduce the following notation: $(a^*)^* = a^{**}$.

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