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MOTION ON TANGENT BUNDLES WHICH PRESERVES BOTH THE ORTHOGONAL AND TANGENT STRUCTURES

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In [1], automorphisms between arbitrary fiber bundles, which preserve the π-structure, were studied; the structure of the curvature and torsion tensors of a fiber bundle admitting maximal group of automorphisms was determined, and automorphisms which also preserve a given linear connection object were considered. In [2], the following three groups of motions in a tangent bundle with the special Sasaki type metric were considered: the group of motions preserving the bundle structure, the group consisting of prolonged transformations, and the group of arbitrary motions. The maximal dimensions of such groups of motions were determined.

In the present article we study the motions of a tangent bundle endowed with an arbitrary Riemannian metric, nondegenerate on fibers, which preserve the orthogonal and tangent structures. It is supposed that the metric connection is canonical and the distributions H and V are J-isometric. All calculations are made in terms of a nonholonomic frame field. We use the apparatus of the theory of automorphisms of fiber bundles, which was developed in [1].

On a tangent bundle TM we consider a Riemannian metric of arbitrary signature, which is nondegenerate on the fibres. Given any (generally speaking, nonholonomic) frame field $\{e_A\}$ adapted to the π-structure and the conjugate coframe field $\{\theta^A\}$: $\theta^A(e_B) = \delta^A_B$, we have

$$d\sigma^2 = G_{AB}\theta^A\theta^B, \tag{1}$$

where

$$G_{AB} = \begin{pmatrix} G_1 & G_4 \\ G_3 & G_2 \end{pmatrix}, \quad G_1 = (G_{ij}), \quad G_2 = (G_{\bar{i}\bar{j}}), \quad G_3 = (G_{\bar{i}j}), \quad G_4 = (G_{i\bar{j}}).$$

Let $v = v^A e_A$ be an arbitrary vector field on TM and

$$X^{A'} = X^A + v^A\tau \tag{2}$$

be the corresponding infinitesimal transformation. Transformation (2) is a motion if it preserves the metric, i. e., the vector field v satisfies the Killing equations

$$\underset{v}{D}G_{AB} = 0. \tag{3}$$

We shall subdivide these equations into the following three groups:

$$\underset{v}{D}G_{ij} = 0, \quad \underset{v}{D}G_{\bar{i}\bar{j}} = 0, \quad \underset{v}{D}G_{\bar{i}j} = 0. \tag{4}$$

Let ∇ be a metric connection with a given torsion tensor (see [3]), i. e., ∇ satisfies the condition $\nabla_C G_{AB} = 0$. Then the Killing equations (3) take the form

$$2v_{(AB)} = v_{AB} + v_{BA} = 0,$$

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