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THE METHOD OF SPLITTING IN THE THEORY OF REGULAR AND SINGULAR PERTURBATIONS

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We suggest a non-traditional spectral method (which can be called also a method of splitting) for solving some classes of problems of the theory of regular and singular perturbations. In contrast to the known method (see [1]–[10]) the suggested one seems to be more convenient for a numerical realization. By applying this method, some new algorithms were obtained for solving the problem on regular perturbation of finite-dimensional operators of both simple and complex Jordan structures, some problems of the theory of branching (see [11]) (not requiring to construct the Newton diagram, see [4], [5]) of singularly perturbed initial and boundary value problems (see [12], [13]) in the presence of exponential and power boundary layers, and also some spectral problems for linear differential operators of arbitrary order (for example, for the Sturm–Liouville and Dirac operators, see [10], which possess application in Quantum Mechanics).

1. Solution of some problems of the theory of regular perturbations and the theory of branching

Consider an algorithm for solving the classical problem of the theory of perturbations in R^n:

$$A(\varepsilon)S_j(\varepsilon) = \lambda_j(\varepsilon)S_j(\varepsilon) \quad \Big(A(\varepsilon) = \sum_{|k|\geq 0} A_k\varepsilon^k, \ \ |\varepsilon| < \varepsilon^0\Big), \tag{$*$}$$

which in contrast to the known one (see [1]–[3]) allows us to determine at once (without constructing the Newton diagram) all the eigenvalues $\lambda_j(\varepsilon)$ and eigenvectors $S_j(\varepsilon)$ with prescribed and uniform with respect to ε accuracy (including the case of a multidimensional perturbation $\varepsilon \in R^m$).

In what follows, for the sake of convenience, for an arbitrary square matrix A we introduce the following notation: $A = \{a_{jk}\}_1^n = \{A_{jk}\}_1^p$, $\overline{A} = \operatorname{diag}\{a_{11}, \dots, a_{nn}\}$, $\overline{\overline{A}} = A - \overline{A}$, $\widehat{A} = \operatorname{diag}\{A_{11}, \dots, A_{pp}\}$, $\hat{\hat{A}} = A - \widehat{A}$ $(2 \leq p < n)$. If the matrix A_0 possesses a simple spectrum $\{\lambda_{0j}\}_1^n$, relation $(*)$ is equivalent to the matrix equation $A(\varepsilon)S(\varepsilon) = S(\varepsilon)\Lambda(\varepsilon)$, where $\Lambda(\varepsilon) = \operatorname{diag}\{\lambda_1(\varepsilon), \dots, \lambda_n(\varepsilon)\}$, and the matrix $S(\varepsilon) = \{S_1(\varepsilon), \dots, S_n(\varepsilon)\}$ consists of eigenvectors of the matrix $A(\varepsilon)$. The change $S(\varepsilon) = S_0H(\varepsilon)$, where $S_0^{-1}A_0S_0 = \Lambda(0) = \Lambda_0$, allows us to pass to the equation

$$B(\varepsilon)H(\varepsilon) = H(\varepsilon)\Lambda(\varepsilon) \quad \Big(B(\varepsilon) = S_0^{-1}A(\varepsilon)S_0 = \Lambda_0 + \sum_{|k|\geq 1} B_k\varepsilon^k\Big), \tag{$**$}$$

whose solution can be represented (see [11]) in the form of converging power series $H(\varepsilon) = E + \sum_{|k|\geq 1} \overline{\overline{H}}_k\varepsilon^k$, $\Lambda(\varepsilon) = \sum_{|k|\geq 0} \Lambda_k\varepsilon^k$. Indeed, by equaling the coefficients in $(**)$ at same powers of ε, we