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Hypercomplex Numbers in the Theory of Physical Structures

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Abstract—Hypercomplex numbers are used for a classification of physical structures.

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A representator and a verificator are basic notions in the theory of physical structures (TPS) ([1], pp. 48–51). A representator is a physical quantity measured in experience. In a mathematical sense, it is a function of two points from a set or from two different sets. A verificator represents a physical law as a functional relation between results of measures, it is written in phenomenologically symmetric form for any cortege containing a finite and fixed number of points from the sets on which the representator defines a physical structure.

One of the main problems of the TPS is that of classification of representators and the corresponding verificators. In the classifications that have already been constructed ([2], pp. 82–83; [3], pp. 113–114), there appear in a natural way hypercomplex numbers of rank 2, not only the standard complex numbers but also double and dual numbers.

As is known, hypercomplex numbers of rank 2 are given by expressions

$$z = x + y\mathbf{i}, \tag{1}$$

where x, y are arbitrary real numbers and $\mathbf{i}$ is so-called imaginary unit. The addition operation is the standard one. The multiplication is related to the addition by the distributivity law and is defined by the square of the imaginary unit:

$$\mathbf{i}^2 = a + b\mathbf{i}, \tag{2}$$

where a and b are real numbers.

The multiplication thus defined is commutative and associative. Passing to another basis, for the square of the imaginary unit, one can obtain, instead of expression (2), three mutually exclusive possibilities ([4], P. 9):

$$\mathbf{i}^2 = -1, +1, 0. \tag{3}$$

Thus, there are three systems of hypercomplex numbers of rank two: the standard complex numbers with $\mathbf{i}^2 = -1$, among which there are no zero divisors; the double numbers with $\mathbf{i}^2 = +1$, which contain the zero divisors $z = x \pm x\mathbf{i}$; the dual numbers with $\mathbf{i}^2 = 0$ and purely imaginary zero divisors $z = y\mathbf{i}$.

In the TPS, hypercomplex numbers of rank two appeared first in the classification of two-dimensional phenomenologically symmetric geometries ([2], § 6) given on a plane M with local coordinates x, y by a metric function (representator)

$$f(ij) = f(x_i, y_i, x_j, y_j), \tag{4}$$

where $\langle ij \rangle \in M^2$. A verificator establishes a relation between six "distances" for any ordered quadruple $\langle ijkl \rangle \in M^4$:

$$\Phi(f(ij), f(ik), f(il), f(jk), f(jl), f(kl)) = 0. \tag{5}$$

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