ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2009, Vol. 53, No. 6, pp. 60–62. © Allerton Press, Inc., 2009.
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Integral Representation of Velocity in a Flow with Separation Simulated by a Vortex Sheet

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Received March 14, 2008

Abstract—The goal of this paper is to present a new integral operator that defines the velocity of a streamline flow in terms of the intensity of a free vortex sheet. This operator is important for numerical simulation of flows.

DOI: 10.3103/S1066369X09060115

Key words and phrases: *separated flow, vortex sheet, integral operator*.

In this paper we give an integral representation of the velocity on a contour that moves in a two-dimensional flow of an incompressible fluid. We simulate the separated boundary layer by a free vortex sheet and replace the contour by a sheet of bound vortices. Such a replacement is a technique commonly used for modeling and computing separated flows of an ideal or low-viscous fluids. In support of this, see the recently published monographs [1, 2].

Let a closed and nonself-intersecting contour Γ be represented in the complex plane z by a periodic function of the class C^2, $z(s) = x(s) + iy(s)$ in a fixed Cartesian system of coordinates, $0 \leq s \leq l$. As the arc-coordinate s increases, the counter is described in the counterclockwise direction.

Consider an unsteady flow over Γ induced by a vortex sheet L located in the flow region. Assume that at a fixed time instant t the sheet L represents a C^2-curve with a continuously distributed on it vorticity of the density $\gamma_L(\sigma; t)$, where σ is the arc-coordinate on L, $0 \leq \sigma \leq l_1(t)$.

We find hydrodynamic properties of the flow by replacing the contour Γ with a bound vortex sheet of strength $\gamma_\Gamma(s; t)$. First we consider the case when the sheet L degenerates to a single vortex of strength γ_a at a point a of the complex z-plane. In this case the distribution of the vorticity along Γ plays the role of a special fundamental solution $\gamma_a(s)$. For an arbitrary sheet L we express $\gamma_\Gamma(s)$ in terms of this solution. (Since the time instant t is fixed, we do not indicate the time dependence of hydrodynamic values.) The impermeability condition on the counter Γ leads us to the following equation for $\gamma_a(s)$:

$$\int_\Gamma \ln|z - \zeta(s)|\nu(s)ds = 2\pi \ln|z - a| + C, \quad z \in \Gamma, \tag{1}$$

where

$$\nu(s) = -\frac{2\pi}{\gamma_a}\gamma_a(s).$$

We determine the unknown constant of integration C from the condition of zero total circulation

$$\int_0^l \gamma_a(s)ds + \gamma_a = 0,$$

which is equivalent to the following requirement to ν:

$$\int_0^l \nu(s)ds = 2\pi. \tag{2}$$

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