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# Motions in Kawaguchi Spaces with Special Metric

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In this paper, we establish the maximum dimension of a group of motions of special Kawaguchi spaces and find invariant characteristic of spaces of maximal mobility.

**1.** A Kawaguchi space is an $n$-dimensional smooth manifold $M$ in which the length $s$ of a curve $x(t)$ is defined by the integral [1]

$$s = \int L(x, \dot{x}, \ddot{x}, \dots, x^{(k)})dt, \tag{1}$$

where $L$ is a scalar function (Lagrangian) of the arguments $x^i$, $\dot{x}^i = \frac{dx^i}{dt}$, $\ddot{x}^i = \frac{d^2x^i}{dt^2}, \dots, x^{(k)^i} = \frac{d^kx^i}{dt^k}$ which are the coordinates of a $k$-velocity of a point $x \in M$. It is assumed that integral (1) takes nonnegative values and does not depend on the parametrization of a curve.

Consider an $n$-dimensional ($n$ is even) Kawaguchi space with metric (1) defined by a Lagrangian of the form [2]

$$L(x, \dot{x}, \ddot{x}) = (\omega_{ij}(x)\dot{x}^i\ddot{x}^j + \gamma_{ijk}(x)\dot{x}^i\dot{x}^j\dot{x}^k)^{\frac{1}{3}}, \tag{2}$$

where $\omega_{ij}$ are the components of a nondegenerate differential 2-form $\omega$ defining an almost symplectic structure on $M$, $\omega_{ij} = -\omega_{ji}$, $\det\|\omega_{ij}\| \neq 0$, and $\gamma_{ijk}$ is a collection of functions symmetric with respect to all indices and defining a differential-geometric object $\gamma$ with the transformation law $\gamma_{i'j'k'} = \gamma_{ijk}\partial_{i'}x^i\partial_{j'}x^j\partial_{k'}x^k + \omega_{pq}\partial_{(i'}x^p\partial^2_{j'k')}x^q$, which guarantees that the Lagrangian $L(x, \dot{x}, \ddot{x})$ is invariant with respect to the coordinate changes $x^{i'} = x^{i'}(x^1, \dots, x^n)$, $\dot{x}^{i'} = \dot{x}^i\partial_i x^{i'}$, $\ddot{x}^{i'} = \ddot{x}^i\partial_i x^{i'} + \dot{x}^i\dot{x}^j\partial^2_{ij}x^{i'}$.

The Zermelo conditions $\dot{x}^i\dot{\partial}_i L + 2\ddot{x}^i\ddot{\partial}_i L = L$, $\dot{x}^i\ddot{\partial}_i L = 0$ for Lagrangian (2) hold identically. Therefore, integral (1) does not depend on the parameterization of a curve ($\partial_i = \frac{\partial}{\partial x^i}$, $\dot{\partial}_i = \frac{\partial}{\partial \dot{x}^i}$, $\ddot{\partial}_i = \frac{\partial}{\partial \ddot{x}^i}$, $i, j, k, \dots = 1, \dots, n$).

**2.** From the Euler–Lagrange equations $\frac{\partial L}{\partial x^i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}^i} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{x}^i} = 0$ it follows that one can associate to Lagrangian (2) the connection $\overset{\circ}{\nabla}$ [1, 3]:

$$\overset{\circ}{\Gamma}{}^k_{ij} = \omega^{kp}\left\{\frac{1}{3}(\partial_i\omega_{pj} + \partial_j\omega_{pi}) + \gamma_{ijp}\right\}. \tag{3}$$

Earlier, studying the geometry of the integral $\int F(x, y, y', y'')$, E. Cartan [4] has constructed a similar connection for $n = 2$.

Connection (3) is torsion-free and, as can be easily verified, has the following property [3, 5]:

$$\overset{\circ}{\nabla}_i\omega_{jk} = (d\omega)_{ijk}, \tag{4}$$

where $(d\omega)_{ijk} = \partial_{[i}\omega_{jk]}$ are the components of the exterior differential $d\omega$ of the fundamental form $\omega$. If $d\omega = 0$, i.e., an almost symplectic structure is a symplectic one, then from (4) it follows that the

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