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SPECTRAL ANALYSIS OF DIFFERENCE OPERATORS WITH STRONGLY CONTINUOUS COEFFICIENTS

A. G. Baskakov

Let G be a locally compact Abelian group (with the algebraic operation written in additive form), $\widehat{G}$ be the dual group of continuous unitary characters of the group G (with a multiplicative form of writing the algebraic operation), X be a complex Banach space, and $\mathcal{F}(G, X)$ be the Banach space of measurable functions which are defined on the group G with values in X (the choice of the space $\mathcal{F}(G, X)$ is to be refined in Section 1). We denote by $\operatorname{End} Y$ the Banach algebra of linear bounded operators acting in the Banach space Y.

In the present article we study the spectral properties of the difference operators $\mathcal{D} \in \operatorname{End} \mathcal{F}(G, X)$ of the form

$$(\mathcal{D}x)(g) = \sum_{n \geq 1} A_n(g) x(g + g_n), \quad x \in \mathcal{F}(G, X), \quad g, g_n \in G, \quad n \geq 1, \tag{1}$$

where the functions A_n, $n \geq 1$ (called coefficients of the operator $\mathcal{D}$), belong to the Banach algebra $C_s(G, \operatorname{End} X)$ of strongly continuous and bounded on G functions with values in $\operatorname{End} X$. In addition, there is assumed that the following condition is fulfilled: $\sum\limits_{n \geq 1} \|A_n\|_\infty < \infty$, where $\|B\|_\infty = \sup\limits_{g \in G} \|B(g)\|$ $\forall B \in C_s(G, \operatorname{End} X)$. The main results of the present article are related to the study of the structure of operators inverse to difference ones, results are obtained with the use of representation of Abelian groups. In Theorem 1 we prove that operators of the view (1) form a filled subalgebra in the Banach algebra $\operatorname{End} \mathcal{F}(G, X)$. It serves as a base for a more deep investigation of spectral (structure) properties of difference operators. A special attention is paid to the difference operators of the form

$$(\mathcal{D}_0 x)(g) = x(g) - B(g) x(g - g_0), \quad x \in \mathcal{F}(G, X), \tag{2}$$

where $g_0 \in G$ and $B \in C_s(G, \operatorname{End} X)$. This is related to the fact that abstract hyperbolic operators are generators of the semigroups of the form (2) (in this case, $G = \mathbb{R}$; see Theorems 4–6). Theorem 6 enables us in many questions to reduce the study of abstract hyperbolic operators to the study of corresponding difference operators. Apparently, this result can be useful in investigation of control problems. Spectral properties of the operators of the form (2) are considered in Theorems 2 and 3.

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