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MODIFIED P-ADIC INTEGRAL AND MODIFIED P-ADIC DERIVATIVE FOR FUNCTIONS DEFINED ON A HALF-AXIS

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Introduction

Let $\mathbf{P} = \{p_j\}_{|j|\in\mathbf{N}}$, where $p_j \in \mathbf{N}$, $2 \le p_j \le N$, $p_{-j} = p_j$ for all $|j|$ from $\mathbf{N}$. Put $m_j = p_1 \cdot \ldots \cdot p_j$ for $j \in \mathbf{N}$, $m_0 = 1$ and $m_{-l} = 1/m_l$ for $l \in \mathbf{N}$. Each $x \in [0;\infty)$ admits the expansion

$$x = \sum_{j=1}^{k(x)} x_{-j} m_{j-1} + \sum_{j=1}^{\infty} x_j/m_j, \tag{1}$$

where $0 \le x_j < p_j$ for $j \in \mathbf{Z}$. It is defined uniquely if for $x = k/m_n$ we consider expansions with a finite number of non-zero x_i. If $n \in \mathbf{N}$ is written in the form $n = \sum\limits_{j=1}^{k(n)} n_j m_{j-1}$ then, by definition, $\chi_n(x) = \prod\limits_{j=1}^{k(n)} \exp(2\pi i n_j x_j/p_j)$. For $n = 0$ we put $\chi_0(x) \equiv 1$. If $y \in [0;\infty)$ has the form

$$y = \sum_{j=1}^{k(y)} y_{-j} m_{j-1} + \sum_{j=1}^{\infty} y_j/m_j, \tag{1'}$$

where $0 \le y_j < p_j$ for $j \in \mathbf{Z}$ then, by definition,

$$x \oplus y = z = \sum_{j=1}^{\max(k(x),k(y))} z_{-j} m_{j-1} + \sum_{j=1}^{\infty} z_j/m_j,$$

where $z_j = x_j + y_j \pmod{p_j}$, $0 \le z_j < p_j$. The operation $x \ominus y$ is defined analogously.

According to (1) and (1′), we define the kernel

$$\chi(x,y) = \exp\left(2\pi i\left(\sum_{j=1}^{k(y)} x_j y_{-j} + \sum_{j=1}^{k(x)} x_{-j} y_j\right)\right),$$

which has the following properties for $x, y, z \in \mathbf{R}_+$:

$$1)\ \ \chi(x,y) = \chi(y,x); \quad |\chi(x,y)| = 1; \tag{2}$$

$$2)\ \ \chi(x,y) = \chi([x],\{y\})\chi([y],\{x\}), \tag{3}$$

where $[x]$ is the entire part of x, and $\{x\}$ is its fractional part (here $\chi(n,y)$, $n \in \mathbf{Z}_+$, coincides with $\chi_n(y)$) on $[0,1)$;

$$3)\ \ \chi(x \oplus z, y) = \chi(x,y)\chi(z,y); \quad \chi(x \ominus z, y) = \chi(x,y)\overline{\chi(z,y)} \tag{4}$$