

A NEW APPROACH TO THE THEORY OF LINEAR PROBLEMS
 FOR SYSTEMS OF DIFFERENTIAL PARTIAL EQUATIONS, III

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In the present article we keep the notation, notions, agreements, and numeration used in the first two parts (see [1], [2]). We shall consider the second application of abstract results obtained earlier.

2. *(b-a)-periodic problem.* In this item $H = L_m^2((a, b))$, φ coincides with (1.4), i. e.,

$$\varphi = \{e(j) \exp(2\pi i k \bullet \frac{x-a}{b-a}), j = 1, \dots, m, k \in \mathbb{Z}^n\}.$$

For the sake of simplicity we assume that, for each $\alpha \in \Phi$, the operator $C_\alpha(x)$ is the operator of multiplication by the matrix $C_\alpha(x)$, which is measurable and square. As for the matrices, they are of the dimension (m, m) , $m < +\infty$.

Theorem 5.2. *Let $C_\alpha(x) \in C_{m,m}^{\{\alpha\}}(b-a) \forall \alpha \in \Phi$, $q(z) = \sum_{\alpha \in \Phi} a_\alpha (iz)^\alpha$ be a scalar polynomial satisfying the conditions*

$$|q(k)|^2 \geq g_1 \sum_{\alpha \in \Phi} \sum_{\beta \leq \alpha} |k^\beta|^2 \quad \text{as soon as } k \in \mathbb{Z}^n, \quad (5.15)$$

for all $\gamma < \alpha$, every $\alpha \in \Phi$, all $|k| \geq N(\varepsilon_1)$, and a certain $\varepsilon_1 > 0$

$$(|k^\gamma|/|q(k)|) < \varepsilon_1, \quad (5.16)$$

moreover, the constants $g_1 > 0$, $N(\varepsilon_1)$ and ε_1 do not depend on k . If ε_1 is sufficiently small, then the following assertions are equivalent:

1. numbers $t \in \mathbb{R}$, $d_1(t)$, $d_2 > 0$, $\alpha_0 \in \mathbb{C}$, exist such that, for each $x \in [a, b]$ and all $z \in \mathbb{C}^m$, $k \in \mathbb{Z}^n$, the bounds are valid

$$\left| \left(\alpha_0 t E + \sum_{\alpha \in \Phi} C_\alpha(x) \left(\frac{2\pi i k}{b-a} \right)^\alpha \right) z \right|^2 \geq |z|^2 (d_1(t) + d_2 |q(k)|^2), \quad (5.17)$$

where $d_1(t) \rightarrow +\infty$ if $|t| \rightarrow +\infty$;

2. numbers $t \in \mathbb{R}$, $d_3(t)$, $d_4 > 0$, $d_5 > 0$, $\alpha_0 \in \mathbb{C}$, exist such that, for all $h(x) \in C_m^{+\infty}(b-a)$, the bounds are valid

$$\begin{aligned} \|(P(x, \partial/\partial x) + \alpha_0 t E)h(x)\| &\geq d_3(t) \|h(x)\| + d_4 \|q(-i\partial/\partial x)h(x)\| \geq \\ &\geq d_3(t) \|h(x)\| + d_5 \sum_{\alpha \in \Phi} \sum_{\beta \leq \alpha} \|h^{(\beta)}(x)\|, \end{aligned} \quad (5.18)$$

where $d_3(t) \rightarrow +\infty$ if $|t| \rightarrow +\infty$.

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