Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 43, No. 11, pp.47–57, 1999 *UDC 517.9*

A NEW APPROACH TO THE THEORY OF LINEAR PROBLEMS FOR SYSTEMS OF DIFFERENTIAL PARTIAL EQUATIONS, III

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In the present article we keep the notation, notions, agreements, and numeration used in the first two parts (see [1], [2]). We shall consider the second application of abstract results obtained earlier.

2. *$(b-a)$-periodic problem.* In this item $H = L^2_m((a,b))$, φ coincides with (1.4), i.e.,

$$\varphi = \{e(j)\exp(2\pi i k \bullet \tfrac{x-a}{b-a},\ j = 1,\dots,m,\ k \in \mathbb{Z}^n\}.$$

For the sake of simplicity we assume that, for each $\alpha \in \Phi$, the operator $C_\alpha(x)$ is the operator of multiplication by the matrix $C_\alpha(x)$, which is measurable and square. As for the matrices, they are of the dimension (m,m), $m < +\infty$.

Theorem 5.2. *Let $C_\alpha(x) \in C^{\{\alpha\}}_{m,m}(b-a)\ \forall\, \alpha \in \Phi$, $q(z) = \sum\limits_{\alpha\in\Phi} a_\alpha (iz)^\alpha$ be a scalar polynomial satisfying the conditions*

$$|q(k)|^2 \ge g_1 \sum_{\alpha\in\Phi}\sum_{\beta\le\alpha} |k^\beta|^2 \quad \textit{as soon as} \quad k \in \mathbb{Z}^n, \tag{5.15}$$

for all $\gamma < \alpha$, every $\alpha \in \Phi$, all $|k| \ge N(\varepsilon_1)$, and a certain $\varepsilon_1 > 0$

$$(|k^\gamma|/|q(k)|) < \varepsilon_1, \tag{5.16}$$

moreover, the constants $g_1 > 0$, $N(\varepsilon_1)$ and ε_1 do not depend on k. If ε_1 is sufficiently small, then the following assertions are equivalent:

1. *numbers $t \in \mathbb{R}$, $d_1(t)$, $d_2 > 0$, $\alpha_0 \in \mathbb{C}$, exist such that, for each $x \in [a,b]$ and all $z \in \mathbb{C}^m$, $k \in \mathbb{Z}^n$, the bounds are valid*

$$\Big|\Big(\alpha_0 tE + \sum_{\alpha\in\Phi} C_\alpha(x)\Big(\frac{2\pi i k}{b-a}\Big)^\alpha\Big)z\Big|^2 \ge |z|^2(d_1(t) + d_2|q(k)|^2), \tag{5.17}$$

where $d_1(t) \to +\infty$ if $|t| \to +\infty$;

2. *numbers $t \in \mathbb{R}$, $d_3(t)$, $d_4 > 0$, $d_5 > 0$, $\alpha_0 \in \mathbb{C}$, exist such that, for all $h(x) \in C^{+\infty}_m(b-a)$, the bounds are valid*

$$\|(P(x,\partial/\partial x) + \alpha_0 tE)h(x)\| \ge d_3(t)\|h(x)\| + d_4\|q(-i\partial/\partial x)h(x)\| \ge$$
$$\ge d_3(t)\|h(x)\| + d_5 \sum_{\alpha\in\Phi}\sum_{\beta\le\alpha} \|h^{(\beta)}(x)\|, \tag{5.18}$$

where $d_3(t) \to +\infty$ if $|t| \to +\infty$.