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Mathematical Modeling of the Stress State of a Transverse Plastic Layer in a Round Rod

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Abstract—We construct mathematical models of plastic deformations of a continuous round rod containing a transverse (less strong) inhomogeneous layer under an axial load. We thoroughly study the local strengthening of such layers by involving the base material of the rod in the plastic deformation process. We obtain explicit formulas for the critical stress states in the layer and the critical axial load on the rod.

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INTRODUCTION

Denotations and conventions. In this paper we use the usual denotations adopted in solid mechanics. We denote by $\varkappa$ the ratio of the layer thickness to the rod diameter; the superscript "MS" ("LS") indicates that a value relates to a more strong (a less strong) part of a joint (to avoid an awkwardness in formulas we sometimes omit the superscript "LS"). We adopt the following abbreviations: mathematical model (models) (MM), stress state (SS), stress-strain state (SSS), less strong (LS), more strong (MS), flat cross-section hypothesis (FCSH), variable separation hypothesis (VSH).

The goal of this paper and the subject of the study. The mentioned subject was studied by many authors, namely, by L. M. Kachanov and O. A. Bakshi [1], their colleagues K. Satoh and M. Toyoda [3], and others. In several papers for simplifying an MM one accepts the hypothesis of full plasticity and the Prandtl hypothesis, according to which the shear stresses are independent of one of two variables. But real distributions of stresses in the LS transverse layer do not agree (to a large extent) with these assumptions. Therefore the development of more adequate models is a problem of today. The SS of a layer essentially depends on mechanical properties of the basic rod material, primarily, on the coefficient K of mechanical heterogeneity, i.e., the ratio of the plastic constant of the basic rod material to that of the layer material. In order to take into account the influence of K one needs to solve the conjunction problem on the contact boundary between the layer and the basic rod material. This problem was not stated (and solved) earlier. The solution given in [1] is obtained under the assumption that plastic deformations take place only in the LS interlayer. For finite (even not too large) values of K that occur in welded joints this assumption leads to a significant overstatement of critical loads.

The goal of this work is to develop an MM of the SS of heterogeneous continuous rods under an axial load and, on this basis, to estimate the influence of mechanical and geometrical parameters on the rod bearing capacity.

The material of the layer and the MS material of the rod (Fig. 1) are assumed to be ideally rigid-plastic, homogenous, and isotropic. The obtained results can be also applied for strengthened materials, because at the moment of the critical load their properties are close to those of ideally plastic materials. By

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