ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2012, Vol. 56, No. 8, pp. 22–27. © Allerton Press, Inc., 2012.
Original Russian Text © N.S. Gabbasov and S.A. Solov'eva, 2012, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2012, No. 8, pp. 27–33.

Special Versions of the Collocation Method for a Class of Integral Equations of the Third Kind

N. S. Gabbasov* **and S. A. Solov'eva****

Naberezhnye Chelny Branch of Kazan (Volga Region) Federal University, pr. Syuyumbike 10a, Naberezhnye Chelny, 423812 Russia

Received June 10, 2011

Abstract—In this paper we propose and justify special direct methods for the approximate solution of equations of the third kind in the space of distributions.

DOI: 10.3103/S1066369X12080038

Keywords and phrases: *third-kind integral equation, space of distributions, approximate solution, theoretical substantiation.*

We consider the following integral equation of the third kind (ETK):

$$(Ax)(t) \equiv (Ux)(t) + (Kx)(t) = y(t), \tag{1}$$

where

$$(Ux)(t) \equiv x(t)t^{p_1}(1-t)^{p_2}\prod_{j=1}^{q}(t-t_j)^{m_j}, \ \ (Kx)(t) \equiv \int_0^1 K(t,s)x(s)ds, \ \ t \in I \equiv [0,1],$$

$p_1, p_2 \in R^+$, $t_j \in (0,1)$, $m_j \in N$ $(j = \overline{1,q})$, K and y are known continuous functions with certain "pointwise smoothness" properties, and x is the desired function. Such equations occur in some problems in the elasticity theory, particles dispersion, neutron transfer (e.g., [1] and references therein; [2]). As a rule, natural classes of solutions to ETK are special spaces of distributions. Since the ETK under consideration can be solved exactly only in some particular cases, both in the theory and in applications one needs approximate solutions methods with a proper theoretical justification. Some relevant results are obtained in papers [3] and [4]. In [3] one proposes and justifies several special direct methods for solving ETK (1). These methods are based on splines of the first and second orders and on polynomials in the space $D\{p_1, p_2; \overline{m}, \overline{\tau}\}$ of distributions and in the V-type space in particular cases of zeros of the coefficient at the desired function outside the integral (in what follows for brevity we just say "the coefficient"). In [4] one considers close issues in the space $V\{p_1, p_2; \overline{m}, \overline{\tau}\}$.

In this paper, using results obtained in [3–6] we develop and justify in the sense of [7] (Chap. 1) special variants of the collocation method for ETK (1) (i.e., in the general case of zeros of the coefficient) based on Bernstein polynomials and Hermite–Fejer interpolation polynomials.

1. Spaces of basic functions and distributions. Let $C \equiv C(I)$ be the space of continuous on $I \equiv [0,1]$ functions with the norm

$$\|\varphi\|_C \equiv \max_{t\in I} |\varphi(t)|.$$

Following [8], we denote by $C\{m; t_0\}$ the class of functions $y(t) \in C$ which at the point $t_0 \in (0,1)$ have the Taylor derivative $y^{\{m\}}(t_0)$ of order m.

*E-mail: gabbasovnazim@rambler.ru.
**E-mail: solovjeva_sa@mail.ru.