Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 45, No. 2, pp. 68–70, 2001 *UDC 517.521*

BASIS ANALOG OF THE GENERALIZED H-FUNCTION

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1. Introduction

In this article we construct a basis analog of the generalized H-function with a kernel depending on a fractional rational function of the q-gamma functions (see [1]–[6]). This analog can include, for example, the I_q-function (see [7]), the H_q-function (see [8], [9]), the G_q-function (see [8], [10]), the I-function (see [11]–[13]), the Fox H-function (see [14]), the Mayer G-function (see [15]). We obtain the sufficient condition for the convergence of the basis analog of the generalized H-function, integral connections, and some formulas.

2. The basis analog of the generalized H-function

Definition (cf. [7]). By the basis analog of the generalized H-function we call the Mellin–Barnes integral

$$I_q(z \mid \theta(s)) \equiv I_q(z \mid r, p_i, \overline{m}_i, \overline{a}_i, \overline{\alpha}_i) = \frac{1}{2\pi\omega}\int_{L_s} \theta(s) z^s ds, \tag{1}$$

where

$$\frac{1}{\theta(s)} = \sin(\pi s)\sum_{i=1}^{r} Q^{p_i}_{\overline{m}_i,\overline{a}_i,\overline{\alpha}_i}(s), \quad r \in N, \quad Q^{p_i}_{\overline{m}_i,\overline{a}_i,\overline{\alpha}_i}(s) = \prod_{k=1}^{p_i} \Gamma_q^{m_{ik}}(A_{ik} + \alpha_{ik}s);$$

the q-gamma function $\Gamma_q(s) = \frac{(q;q)_\infty}{(q^s;q)_\infty}(1-q)^{1-s}$ is defined in [1], [2], $(a,q)_\infty = \prod\limits_{n=0}^{\infty}(1-aq^n)$, $|q|<1$, $A_{ik} = \frac{1}{2} - (a_{ik} - \frac{1}{2})\operatorname{sign}\alpha_{ik}$, p_i is a natural number, $i = \overline{1,r}$, $k = \overline{1,p_i}$; $\overline{m}_i = (m_{i1}, m_{i2}, \dots, m_{ip_i})$, m_{ik} are integer numbers; $\overline{a}_i = (a_{i1}, a_{i2}, \dots, a_{ip_i})$, a_{ik} are complex; $\overline{\alpha}_i = (\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{ip_i})$, α_{ik} are real numbers; z^s is defined in [15], $\omega = (-1)^{1/2}$. The contour of integration L_s goes from $-i\infty$ to $i\infty$ so that the poles of the functions $\Gamma_q^{m_{ik}}(A_{ik} + \alpha_{ik}s)$ with $m_{ik} < 0$, $\alpha_{ik} < 0$ lie to the right from the contour and the poles of the functions $\Gamma_q^{m_{ik}}(A_{ik} + \alpha_{ik}s)$ with $m_{ik} < 0$, $\alpha_{ik} > 0$, $i = \overline{1,r}$, $k = \overline{1,p_i}$, lie to the left and are situated at least at a certain distance $\varepsilon > 0$ from the contour.

Theorem 1 (cf. [7]). *The basis analog of the generalized H-function* (1) *converges if* $|\arg z| < \pi$. *If, in addition,* $m_{ik} = -1$ *for* $i = \overline{1,r}$, $k = \overline{1,m+n}$, $a_{ik} = a_k$, $\alpha_{ik} = \alpha_k > 0$, $k = \overline{1,n}$; $a_{ik} = b_{k-n}$, $\alpha_{ik} = -\beta_{k-n} < 0$, $k = \overline{n+1,n+m}$; $m_{ik} = 1$ *for* $i = \overline{1,r}$, $k = \overline{m+n+1, A_i+B_i+2}$; $a_{ik} = a_{i(k-m)}$, $\alpha_{ik} = -\alpha_{i(k-m)} < 0$, $k = \overline{m+n+1, m+A_i}$, $a_{i(A_i+m+1)} = 1$, $\alpha_{i(A_i+m+1)} = -1$; $a_{ik} = b_{i(k-A_i-1)}$, $\alpha_{ik} = \beta_{i(k-A_i-1)} > 0$, $k = \overline{A_i+m+2, A_i+B_i+1}$; $a_{i(A_i+B_i+2)} = 1$, $\alpha_{i(A_i+B_i+2)} = 1$, $p_i = A_i + B_i + 2$, *then the equality is valid*:

$$\lim_{q\to 1_-} I_q(z \mid r, p_i, \overline{m}_i, \overline{a}_i, \overline{\alpha}_i) = I^{m,n}_{A_i,B_i}\left[z \left| \begin{matrix}(a_j,\alpha_i)_{1,n}, \dots, (a_{ij},\alpha_{ij})_{n+1,A} \\ (b_j,\beta_j)_{1,m}, \dots, (b_{ij},\beta_{ij})_{m+1,B_i}\end{matrix}\right.\right],$$

where the function standing in the right side was introduced in [11].