ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2013, Vol. 57, No. 10, pp. 59–61. © Allerton Press, Inc., 2013.
Original Russian Text © A.G. Gein and M.P. Shushpanov, 2013, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2013, No. 10, pp. 69–72.

Defining Relations of a Free Modular Lattice of Rank 3

A. G. Gein* **and M. P. Shushpanov****
(Submitted by L.N. Shevrin)
Ural Federal University, ul. Mira 19, Ekaterinburg, 620002 Russia

Received February 11, 2013

Abstract—For a 3-generated free modular lattice we obtain a set of 11 defining relations and prove that this set is minimal.

DOI: 10.3103/S1066369X13100071

Keywords and phrases: *free modular lattices, defining relations*.

Recall that the *rank* of free algebra from some manifold is the cardinal number of the set of its free generators. We concentrate our attention on a free lattice of the rank 3 in the manifold of modular lattices; we denote it by A. Let F be a free lattice of the rank 3 in the manifold of all lattices; let f, g, and h be its free generators, and let φ be a homomorphism from F to A. By standard considerations of a universal algebra, elements $a = \varphi(f)$, $b = \varphi(g)$, and $c = \varphi(h)$ are free generators of the lattice A. Relations defining this lattice in the manifold of all lattices were considered in [1] and [2]. In the paper [1] one has particularly shown that A can be defined by 21 relations. In [2] one has proved that this set of defining relations is not minimal; namely, it was shown there that 15 relations among those mentioned above define the lattice A, moreover, they form the minimal set of defining relations for A. Note that in [1] one has described a set of seven defining relations for a free distributive lattice of the rank 3, and in [3] this set was proved to be minimal.

The following assertion is the main result of this paper: There exists a set of 11 defining relations for the lattice A. Note that this set is not a subset of the set of defining relations indicated in [1]. Let us enumerate these relations:

$$(a \vee (b \wedge c)) \wedge (b \vee c) = (a \wedge (b \vee c)) \vee (b \wedge c), \tag{1}$$

$$(b \vee (c \wedge a)) \wedge (c \vee a) = (b \wedge (c \vee a)) \vee (c \wedge a), \tag{2}$$

$$(c \vee (a \wedge b)) \wedge (a \vee b) = (c \wedge (a \vee b)) \vee (a \wedge b), \tag{3}$$

$$(a \vee b) \wedge (a \vee c) \wedge (b \vee c) = ((a \wedge (b \vee c)) \vee ((a \vee b) \wedge c)) \wedge ((b \wedge (a \vee c)) \vee ((b \vee a) \wedge c)) \wedge ((a \wedge (c \vee b)) \vee ((a \vee c) \wedge b)), \tag{4}$$

$$(a \wedge b) \vee (a \wedge c) \vee (b \wedge c) = ((a \vee (b \wedge c)) \wedge ((a \wedge b) \vee c)) \vee ((b \vee (a \wedge c)) \wedge ((b \wedge a) \vee c)) \vee ((a \vee (c \wedge b)) \wedge ((a \wedge c) \vee b)), \tag{5}$$

$$(a \vee b) \wedge (a \vee c) = a \vee ((a \vee b) \wedge (a \vee c) \wedge (b \vee c)), \tag{6}$$

$$(b \vee a) \wedge (b \vee c) = b \vee ((a \vee b) \wedge (a \vee c) \wedge (b \vee c)), \tag{7}$$

$$(c \vee a) \wedge (c \vee b) = c \vee ((a \vee b) \wedge (a \vee c) \wedge (b \vee c)), \tag{8}$$

$$(a \wedge b) \vee (a \wedge c) = a \wedge ((a \wedge b) \vee (a \wedge c) \vee (b \wedge c)), \tag{9}$$

$$(b \wedge a) \vee (b \wedge c) = b \wedge ((a \wedge b) \vee (a \wedge c) \vee (b \wedge c)), \tag{10}$$

*E-mail: Alexander.Gein@usu.ru.
**E-mail: Mikhail.Shushpanov@gmail.com.