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Construction of an Asymptotically Periodic Solution to One Nonstationary System with Delay

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Abstract—In this paper we consider an ensemble of two subsystems of linear differential equations with constant delay. One of the subsystems contains an exponential multiplier in its right-hand side. We construct an asymptotically periodic solution of this system.

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We consider the following nonstationary system with constant delay:

$$\begin{aligned} dx(t)/dt &= (A_1 + \varepsilon\overline{S}_1(t))x(t) + (A_2 + \varepsilon\overline{S}_2(t))x(t-\tau) \\ &\quad + (B_1 + \varepsilon\overline{P}_1(t))y(t) + (B_2 + \varepsilon\overline{P}_2(t))y(t-\tau) + \overline{f}_1(t), \\ dy(t)/dt &= e^t[(A_3 + \varepsilon\overline{S}_3(t))x(t) + (A_4 + \varepsilon\overline{S}_4)x(t-\tau) + (B_3 + \varepsilon\overline{P}_3(t))y(t) \\ &\quad + (B_4 + \varepsilon\overline{P}_4(t))y(t-\tau) + \overline{f}_2(t)], \quad t \ge t_0 > 0, \quad \tau = \text{const}, \qquad \tau > 0. \end{aligned} \tag{1}$$

Here $\overline{S}_j$ and $\overline{P}_j$ $(j = 1,2,3,4)$ are periodic (with the period τ) continuously differentiable $m \times m$ matrices, $\max\{\|\overline{S}_j\|, \|\overline{P}_j\|\} \le \overline{P}$, where $\overline{P}$ is a positive constant, ε is a small positive value. Evidently, with $\varepsilon = 0$ we have a linear heterogeneous system

$$\begin{aligned} dx(t)/dt &= A_1x(t) + A_2x(t-\tau) + B_1y(t) + B_2y(t-\tau) + \overline{f}_1(t), \\ dy(t)/dt &= e^t(A_3x(t) + A_4x(t-\tau) + B_3y(t) + B_4y(t-\tau) + \overline{f}_2(t)), \quad t \ge t_0 > 0. \end{aligned} \tag{2}$$

Here A_r, B_r $(r = 1,2,3,4)$ are constant $m \times m$ matrices; $x(t)$, $y(t)$ are m-dimensional vector functions with respect to time (the argument) t; m-dimensional vector functions $\overline{f}_j(t)$ $(j = 1,2)$ are continuously differentiable and periodic (with the period τ).

With $t_0 - \tau \le \eta \le t_0$ a solution to system (1) is defined by the initial vector function $\phi^\top(\eta) = \{\phi_1(\eta), (\phi_2(\eta))\} : x(\eta) = \phi_1(\eta), y(\eta) = \phi_2(\eta)$ (the symbol $\top$ means the vector transposition). Note that system (1) is "perturbed" with respect to system (2), and the perturbances are small due to the smallness of ε. Let us first consider some properties of the unperturbed system (2) (we assume that both a solution to this "unperturbed" system and a solution to the initial system (1) are defined at the time moment t_0 by one and the same initial vector function $\phi(\eta)$).

System (2) is a finite system of differential equations with delay on an infinite time interval. Let us proceed from it to a countable system of ordinary differential equations defined on a finite time period $[0,\tau]$. To this end, put $x_{n+1}(t) = x(t_0 + n\tau + t)$, $y_{n+1}(t) = y(t_0 + n\tau + t)$, $f_r(t) = \overline{f}_r(t_0 + t) : t \in [0,\tau]$ $(r = 1,2)$. We obtain the ensemble of two subsystems of the mth order

$$dx_{n+1}(t)/dt = A_1x_{n+1}(t) + A_2x_n(t) + B_1y_{n+1}(t) + B_2y_n(t) + f_1(t), \tag{3}$$

$$\varepsilon_n dy_{n+1}(t)/dt = e^t(A_3x_{n+1}(t) + A_4x_n(t) + B_3y_{n+1}(t) + B_4y_n(t) + f_2(t)), \tag{4}$$

$$0 \le t \le \tau, \quad \varepsilon_n = e^{-(t_0+n\tau)}$$

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