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ON THE EQUATIONS OF DYNAMICS OF THE NON-NEWTON LIQUID WHICH MOVES WITH SLIDING ALONG BED

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1. In the present article we study the equation

$$u_t = (u^2\sigma^n + \varepsilon \cdot k(u,\sigma))_x, \quad \sigma = uu_x \tag{1.1}$$

by means of the methods of the approximate group analysis.

Equations of the form (1.1) arise, for example, in simulating one-dimensional flow of a film of non-Newton liquid along a horizontal bed with sliding. In this case, the unknown function $u(t,x)$ is the thickness of the film at a point x at the moment t, n is the exponent in rheological law, the second addend in the right-hand side of (1.1) describes the sliding effect, ε is a parameter which often can be considered small (in sliding problems in dimensionless equation (1.1) we have $\varepsilon \approx 0.15$). Exact symmetries and invariant solution of equation (1.1) for $\varepsilon = 0$ (i.e., in absence of sliding) were investigated in [1]–[3] for both one-dimensional case and two-dimensional problems with circular symmetry. Methods which we apply in the present article are based, as in [1]–[3], on the classical group analysis (see [4]) because our investigation is restricted to point groups. This is intrinsic for the class of equations under consideration since even for heat conduction equation with a source its Lie–Becklund group (see [5]) is as a rule (with two exceptions) trivial (see [6]). Their group analysis is reduced to classification of point symmetries (the Lie groups) which was done for nonlinear heat conduction equation without source by L.V. Ovsyannikov (see [7]), and in presence of a source — by V.A. Dorodnitsyn (see [8]). The present article can be considered as a continuation of these works of studying approximate symmetries with a small parameter, their general theory was constructed in [9], [10]. Since equation (1.1) models the flow only up to $O(\varepsilon^2)$, it is natural to consider its approximate symmetries with the same accuracy. The classification of equations of the form (1.1), which admit a group of approximate symmetries, is given in Sections 2 and 3. In Section 4 we give approximate-invariant solution with the accuracy $o(\varepsilon^2)$ for equations obtained in Sections 2 and 3. These equations are of the most interest in the dynamics of non-Newton liquids. For the sake of brevity, further we use the following notation

$$t = y_1, \quad x = y_2, \quad u = y_3, \quad \sigma = y_4, \quad u_t = y_5, \quad u_x = y_6, \quad \sigma_t = y_7, \quad \sigma_x = y_8$$

and then rewrite equation (1.1) into the form

$$F_0^1 + \varepsilon F_1^1 = 0, \quad F_0^2 + \varepsilon F_1^2 = 0, \tag{1.2}$$

where

$$F_0^1 = 2y_3y_6y_4^n + ny_3^2y_4^{n-1}y_8 - y_5, \quad F_0^2 = y_3y_6 - y_4,$$
$$F_1^1 = \frac{\partial k}{\partial y_3}y_6 + \frac{\partial k}{\partial y_4}y_8, \quad F_1^2 \equiv 0.$$

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