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HYPERBANDS OF CURVATURE WITH PARALLEL THIRD FUNDAMENTAL FORM IN EUCLIDEAN SPACE

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1. Basic definitions and main results

The notion of a band in the three-dimensional Euclidean space E_3 was introduced by W.Blaschke [1]. He defined the band to be a curve in E_3 endowed by a smooth field of tangent planes. V.V. Vagner generalized this notion to an m-dimensional surface in an n-dimensional center-affine space [2]

Definition 1. A smooth m-dimensional hyperband H_m in an n-dimensional Euclidean space E_n is an m-parametric manifold of plane elements (x, ξ), where x runs through a surface M_m, and the hyperplane $\xi(x)$ is tangent to M_m at $x \in M_m$ [2].

The surface M_m is called the *basic* surface of the hyperband H_m, and the hyperplanes $\xi(x)$ are called principal tangent hyperplanes of H_m.

For an m-dimensional manifold of hyperplanes in E_n, the manifold of characteristic planes is defined. These planes determine limits of $(n-m-1)$-dimensional intersections of $m+1$ infinitely approaching independent hyperplanes of this manifold. If $(n-m-1)$-dimensional characteristic planes $\Pi(x)$ of the family of principal tangent hyperplanes of a hyperband H_m do not contain directions tangent to the basic surface of H_m, then H_m is said to be regular [2].

If each tangent plane $T_x(M_m)$ of the basic surface M_m is completely orthogonal to the characteristic plane $\Pi(x)$, $x \in M_m$, then H_m is called the hyperband of curvature [3]. For $m = 1$, $n = 3$ the hyperband of curvature was defined in [1].

Definition 2. A surface M_m in E_n is called dual-normalized if there exists an m-dimensional manifold of hyperplanes $\xi(x)$ tangent to M_m such that at any point $x \in M_m$ the $(n-m-1)$-dimensional characteristics $\Pi(x)$ lie in the $(n-m)$-plane $N(M_m)$ normal to M_m [4].

For any point $x \in M_m$, let us take p ($p \leqslant n-m$) directions the $(n-m)$-dimensional normal plane $N(M_m)$ of M_m, and assume that they determine a smooth p-direction field $\nu_p \subset N(M_m)$.

Definition 3. A field ν_p is said to be parallel with respect to the normal connection if, under any movement of an arbitrary point $x \in M_m$, the p-direction $\nu_p(x)$ moves along the $(m+p)$-dimensional plane spanned by the tangent plane $T_x(M_m)$, $x \in M_m$, and the normal plane $\nu_p(x)$ [4].

We say that the principal third fundamental form α_3 (see [4]) of a hyperband is parallel if $\overline{\nabla} h_{ijk} = 0$ [5], where $\overline{\nabla}$ is the Van der Warden-Bortolotti connection. The local structure of hyperband of curvature was studied in [3], [6]. In the present paper we proceed with investigation of hyperband of curvature with parallel principal third fundamental form. Our main results are: